
Easy Units Conversion Methodology

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Units conversion is often necessary in calculations as many types of units were frequently followed to represent the same quantity. It is sometimes confusing to students to make this conversion from one unit to another. A simple methodology that handles units along with the numbers helps to eliminate the confusion. This method can be easily extended logically to most common units conversion situations. This paper introduces the principle of the method, demonstrates the methodology using several examples, outlines the limitations, and informs of the online units conversion tools available to users.

1 Introduction

Often a physical quantity (represented by a combination of number and unit) needs to be converted from one unit to other to express it in other forms. For example, in US, mass is commonly expressed in pounds (lb) while in kilograms (kg) internationally; similarly the liquid volume in gallon (gal) as liter (L). Although unit conversions in the form of relationships, charts, and online resources are available, it is useful to understand the logic so that it is possible to convert from one to any another relevant unit as required.

Sometimes it causes confusion in the mind as to “Whether to multiply or divide?” especially when one knows the conversion formula and works only with numbers, without considering the units in combination. The drawback in working only with numbers is it will not offer any way of checking the correctness of calculation. One simple way of performing correct conversion is “factor-label” or “unit-factor” method (Wikipedia, 2013).

This simple methodology of using the units along with the numbers offers a way of check and guide the units conversion process. Thus the process of units conversion should be performed along with the “units.” The purpose of this paper is to introduce the methodology, demonstrate the methodology with numerous examples from simple to advanced conversion, outline the limitations, and point out the availability of other resources of units conversion.

2 Principle of Methodology

It should also be understood that only a “number” does not make sense unless accompanied by “units” (e.g. as a physical quantity 8 lb makes more sense than simply 8). Let us consider the following mathematical expression:

$$A = B \quad (1)$$

It follows from the above equation that:

$$\frac{A}{B} = 1 \quad \text{or} \quad \frac{B}{A} = 1 \quad (2)$$

Which means that the ratios (A/B or B/A), being unity, when multiplied or divided with any other quantity will not change the value of that quantity. Extending the analogy to product of two quantities, using Eqn. (1) and Eqn. (2):

$$A \times P = B \times Q$$
$$\frac{A \times P}{B \times Q} = 1 \quad \text{or} \quad \frac{B \times Q}{A \times P} = 1 \quad (3)$$

This analogy (Eqn. (3)) can be applied any physical quantity represented by a combination of a number

and an unit, as the number and its unit were considered as a multiplication (product) of these items. As an example, for a common physical quantity we have:

$$\begin{aligned} 1 \text{ lb} &= 0.454 \text{ kg} \\ \frac{1}{0.454} \frac{\text{lb}}{\text{kg}} &= \frac{0.454 \text{ kg}}{1 \text{ lb}} = 1 \end{aligned} \quad (4)$$

Thus, this unity (Eqn. (4)) represented by two numbers and their corresponding units, when multiplied or divided to any other number and its unit will not change its value of quantity. This logic forms the main principle of the units conversion methodology discussed in the work. As the principle applied is the same, the methodology can be extended to other simple or compounds units readily that are not discussed in this work.

It is also necessary to know some of the common conversion factors to perform the conversion. These units conversion formulas are available through several standard sources (e.g. textbook appendices).

3 Useful Common Conversions

Some conversion formulas frequently used are presented in Table 1. It is not necessary to remember or have the two-way conversion formulas (lb ⇒ kg; and kg ⇒ lb), as in the conversion methodology one conversion formula is sufficient.

4 Simple Conversion Examples

Example 4.1

What is the equivalent of 50 pounds (lb) in kilograms (kg)?

The conversion formula known is: 1 lb = 0.454 kg. Following the method outlined earlier (Eqn. (4)), the combination ratio (numbers and units) from the conversion formula can be multiplied with the physical quantity to be converted, which cancels relevant units, and arrive at the results.

Now 50 lb when multiplied with the ratio of unity:

$$50 \text{ lb} = 50 \text{ lb} \times \frac{0.454 \text{ kg}}{1 \text{ lb}}$$

Upon canceling the relevant units and multiplying the numbers:

$$50 \cancel{\text{ lb}} \times \frac{0.454 \text{ kg}}{1 \cancel{\text{ lb}}} = 50 \times 0.454 \text{ kg} = 22.7 \text{ kg} \quad (5)$$

In Eqn. (5), correctly using the second ratio of unity Eqn. (4) the final unit was worked out as “kg.”

Caution: Had the first ratio of unity (1 lb/0.454 kg) used will lead to:

$$50 \text{ lb} \times \frac{1}{0.454} \frac{\text{lb}}{\text{kg}} = \frac{50}{0.454} \frac{\text{lb}^2}{\text{kg}} = 110.13 \frac{\text{lb}^2}{\text{kg}}$$

Even though the number “110.13” did not provide any way of cross checking, the resulting unit of “lb²/kg” is clearly not intended - making the conversion incorrect.

Table 1: Common conversions

Physical-quantity	Basic-unit	Conversion
Mass:	1 lb	= 0.454 kg
	1 lb	= 16 ounce
	1 kg	= 2.205 lb
	1 tonne	= 1000 kg
	1 kg	= 1000 g
Length:	1 m	= 3.28 ft
	1 mile	= 1.609 km
	1 in	= 25.4 mm
	1 m	= 1000 mm
	1 yard	= 0.914 m
Area:	1 mile ²	= 640 acre
	1 acre	= 43 560 foot ²
	1 ha	= 2.471 acre
	1 ha	= 10 000 m ²
Volume:	1 gal	= 3.785 liter
	1 ft ³	= 28.32 liter
	1 bushel	= 8 gal
	1 m ³	= 1000 liter
Pressure:	1 bar	= 100 kPa
	1 psi	= 6 895 Pa
	1 kPa	= 1000 N/m ²
Energy:	1 Btu	= 1055 J
	1 cal	= 4.184 J
	1 kWh	= 3.6 MJ
Power:	1 hp	= 0.746 kW
	1 kW	= 1000 J/s
Time:	1 h	= 60 min
	1 min	= 60 s
Prefixes: kilo = 1 000; mega = 1 000 000 milli = 1/1000; centi = 1/100		

Example 4.2

How many pounds will be 100 kg?

Using the conversion formula and appropriate ratio of unity following Eqn. (4) it follows:

$$100 \cancel{\text{kg}} \times \frac{1 \text{ lb}}{0.454 \cancel{\text{kg}}} = \frac{100}{0.454} \text{ lb} = 220.26 \text{ lb}$$

5 Advanced Conversion Examples

Example 5.1

What will be the area of 200 acres (a) in square miles and (b) square meters?

Using the relevant conversion formulas (Table 1):
(a) In square miles:

$$200 \cancel{\text{acre}} \times \frac{1 \text{ mile}^2}{640 \cancel{\text{acre}}} = \frac{200}{640} \text{ mile}^2 = 0.3125 \text{ mile}^2$$

(b) In square meters:

$$\begin{aligned} 200 \cancel{\text{acre}} \times \frac{1 \cancel{\text{ha}}}{2.471 \cancel{\text{acre}}} \times \frac{10000 \text{ m}^2}{1 \cancel{\text{ha}}} \\ = \frac{200 \times 10000}{2.471} \text{ m}^2 = 809,389 \text{ m}^2 \end{aligned}$$

It can be seen in the above calculations two formulas were used in succession ($\text{acre} \Rightarrow \text{ha} \Rightarrow \text{m}^2$) to arrive at the final conversion. It is also possible to obtain direct conversion from acre to m^2 .

Such “chain-conversion” can be used in advanced conversions using a set of simple basic unit conversion formulas. This methodology can be applied to any compound units - even compound hypothetical units involving several basic units. It is also possible to perform conversion on compound units that are not available from any online resources.

Example 5.2

What will be the area of 100 ft^2 in m^2 ?

It is possible to use the simple conversion formula between feet and meter ($1 \text{ m} = 3.28 \text{ ft}$) to arrive at the areas in square units. The ratio of number and units combination will be again unity when the ratio was squared as follows:

$$\frac{1 \text{ m}}{3.28 \text{ ft}} = \left(\frac{1 \text{ m}}{3.28 \text{ ft}} \right)^2 = \frac{1^2 \text{ m}^2}{3.28^2 \text{ ft}^2} = 1 \quad (6)$$

It should be noted that all the items (numbers and units) should be squared individually in Eqn. (6). Using the Eqn. (6), 100 ft^2 will be converted as:

$$100 \cancel{\text{ft}^2} \times \frac{1^2 \text{ m}^2}{3.28^2 \cancel{\text{ft}^2}} = \frac{100 \times 1^2}{3.28^2} \text{ m}^2 = 9.295 \text{ m}^2$$

Example 5.3

Using the simple conversion of $1 \text{ in} = 25.4 \text{ mm}$, what will be the volume of 24,000 cubic mm in cubic in?

Extending the squaring method used earlier (Example 5.2, Eqn. (6)), volumes can be converted by cubing the ratio of unity as follows:

$$\begin{aligned} 24000 \text{ mm}^3 \times \left(\frac{1 \text{ in}}{25.4 \text{ mm}} \right)^3 \\ = 24000 \cancel{\text{mm}^3} \times \frac{1^3 \text{ in}^3}{25.4^3 \cancel{\text{mm}^3}} \\ = \frac{24000}{25.4^3} \text{ in}^3 = 1.465 \text{ in}^3 \end{aligned}$$

Example 5.4

What will be the speed of 75 mph in (a) kmph and (b) meter per minute?

Using simple conversion between mile and kilometer, time conversion formulas (Table 1), and chain-conversion method performs the required conversion.

(a) In kmph:

$$\begin{aligned} 75 \frac{\cancel{\text{mile}}}{\text{h}} \times \frac{1.609 \text{ km}}{1 \cancel{\text{mile}}} = 75 \times 1.609 \frac{\text{km}}{\text{h}} \\ = 120.68 \text{ kmph} \end{aligned}$$

(b) In meter per minute:

$$\begin{aligned} 75 \frac{\cancel{\text{mile}}}{\cancel{\text{h}}} \times \frac{1.609 \cancel{\text{km}}}{1 \cancel{\text{mile}}} \times \frac{1000 \text{ m}}{1 \cancel{\text{km}}} \times \frac{1 \cancel{\text{h}}}{60 \text{ min}} \\ = \frac{75 \times 1.609 \times 1000}{60} \frac{\text{m}}{\text{min}} = 2011.25 \text{ m/min} \end{aligned}$$

Example 5.5

For a wheat variety the seed rate followed is 90 lb/acre. What will be this seed rate in kg/ha?

Using conversions formulas (Table 1) the conversion

is obtained as follows:

$$90 \frac{\cancel{\text{lb}}}{\cancel{\text{acre}}} \times \frac{1}{2.205} \frac{\text{kg}}{\cancel{\text{lb}}} \times \frac{2.471}{1} \frac{\cancel{\text{acre}}}{\text{ha}}$$

$$= \frac{90 \times 2.471}{2.205} \frac{\text{kg}}{\text{ha}} = 100.86 \text{ kg/ha}$$

Example 5.6

A vehicle offers 31 miles per gallon (mpg) mileage. What will be the mileage in km per liter (kmpl)?

Using conversions formulas (Table 1) the conversion is obtained as follows:

$$31 \frac{\cancel{\text{mile}}}{\cancel{\text{gal}}} \times \frac{1.609}{1} \frac{\text{km}}{\cancel{\text{mile}}} \times \frac{1}{3.785} \frac{\cancel{\text{gal}}}{\text{liter}}$$

$$= \frac{31 \times 1.609}{3.785} \frac{\text{km}}{\text{liter}} = 13.18 \text{ kmpl}$$

Example 5.7

What will be the equivalent of 800 Btu in calories (cal)?

From conversions formulas (Table 1), we have Btu and cal connected by joules (J). Thus applying chain-conversion method the result will be:

$$800 \frac{\cancel{\text{Btu}}}{1} \times \frac{1055}{\cancel{\text{Btu}}} \frac{\text{J}}{1} \times \frac{1}{4.184} \frac{\text{cal}}{\cancel{\text{J}}}$$

$$= \frac{800 \times 1055}{4.184} \text{ cal} = 201,771 \text{ cal}$$

Also:

$$= 201,771 \cancel{\text{cal}} \times \frac{1}{1000} \frac{\text{kcal}}{\cancel{\text{cal}}} = 201.8 \text{ kcal}$$

Example 5.8

What will be 120 kilowatt (kW) in (a) joule/second (J/s), (b) kilocalorie/hour (kcal/h), and (c) horsepower (hp)?

From conversions formulas (Table 1) the conversions are obtained as follows:

(a) In J/s:

$$120 \cancel{\text{kW}} \times \frac{1000}{1} \frac{(\text{J/s})}{\cancel{\text{kW}}} = 120 \times 1000 \text{ (J/s)}$$

$$= 120,000 \text{ J/s}$$

(b) In kcal/h:

$$120 \cancel{\text{kW}} \times \frac{1000}{1} \frac{\cancel{\text{J}}}{\cancel{\text{kW}}} \times \frac{1}{4.184} \frac{\text{cal}}{\cancel{\text{J}}}$$

$$= \frac{120 \times 1000}{4.184} \frac{\cancel{\text{cal}}}{\cancel{\text{h}}} \times \frac{1}{1000} \times \frac{\text{kcal}}{\cancel{\text{cal}}} \times \frac{3600}{1} \times \frac{\cancel{\text{h}}}{\cancel{\text{h}}}$$

$$= \frac{120 \times 1000 \times 3600}{4.184 \times 1000} \frac{\text{kcal}}{\text{h}} = 103,250 \text{ kcal/h}$$

(c) In hp:

$$120 \cancel{\text{kW}} \times \frac{1}{0.746} \frac{\text{hp}}{\cancel{\text{kW}}} = \frac{120}{0.746} \text{ hp}$$

$$= 168.9 \text{ hp}$$

Example 5.9

Express the calorific value of a biomass of 16.93 megajoules/kilogram (MJ/kg) in (a) and British thermal unit/pound (Btu/lb) and (b) kilocalorie/kilogram (kcal/kg).

From conversions formulas (Table 1) we obtain the following conversions:

(a) In BTU/lb:

$$16.93 \frac{\cancel{\text{MJ}}}{\cancel{\text{kg}}} \times \frac{1 \times 10^6}{1} \frac{\cancel{\text{J}}}{\cancel{\text{MJ}}} \times \frac{1}{1055} \frac{\text{Btu}}{\cancel{\text{J}}} \times \frac{1}{2.205} \frac{\cancel{\text{kg}}}{\text{lb}}$$

$$= \frac{16.93 \times 1000,000}{1055 \times 2.205} \frac{\text{Btu}}{\text{lb}} = 7,278 \text{ Btu/lb}$$

(b) In kcal/kg:

$$16.93 \frac{\cancel{\text{MJ}}}{\cancel{\text{kg}}} \times \frac{1 \times 10^6}{1} \frac{\cancel{\text{J}}}{\cancel{\text{MJ}}} \times \frac{1}{4.184} \frac{\cancel{\text{cal}}}{\cancel{\text{J}}} \times \frac{1}{1000} \times \frac{\text{kcal}}{\cancel{\text{cal}}}$$

$$= \frac{16.93 \times 1000,000}{4.184 \times 1000} \frac{\text{kcal}}{\text{kg}} = 4,046 \text{ kcal/kg}$$

6 Limitations

Unit conversion methodology demonstrated thus far will work only when the units converted are proportional, and both units vary linearly starting from zero. Most of the units follow this principle making this method valid for wide variety of applications.

One familiar exception is the temperature units, namely, Fahrenheit (°F), Celsius (°C), and Kelvin. A temperature of 0°C refers to 32°F or 273 K, as opposed to proportional units when one is zero the other will be zero as well (e.g. 0 ft = 0 mile = 0 m). Thus, the temperature units have offsets -

meaning with inter-conversions both units do not start from zero. Therefore, a single constant cannot make the conversion, as seen in the following temperature conversion formulas:

$$^{\circ}\text{F} = 1.8^{\circ}\text{C} + 32 \quad (7)$$

$$^{\circ}\text{C} = (^{\circ}\text{F} - 32)/1.8$$

$$\text{K} = ^{\circ}\text{C} + 273.15 \quad (8)$$

One can see from the above conversion formulas that an addition or subtraction operation is involved and these conversions have two constants. For inter-conversion of temperature units, these formulas involving offsets should be used.

For compound units involving temperatures, the offsets ('32' in Eqn. (7) and '273.15' in Eqn. (8)) can be ignored as shown subsequently. Let F_{ini} be the initial and F_{fin} be the final temperature in $^{\circ}\text{F}$, and C_{ini} and C_{fin} are the corresponding temperatures in $^{\circ}\text{C}$. Substituting these in Eqn. (7) we get:

$$F_{\text{ini}} = 1.8 \times C_{\text{ini}} + 32$$

$$F_{\text{fin}} = 1.8 \times C_{\text{fin}} + 32$$

Subtracting the first relation from the second.

$$(F_{\text{fin}} - F_{\text{ini}}) = 1.8 \times (C_{\text{fin}} - C_{\text{ini}}) + (\cancel{32} - \cancel{32})$$

$$F_{\text{diff}} = 1.8 \times C_{\text{diff}} \quad (9)$$

It can be seen that the "difference in temperatures" in $^{\circ}\text{F}$ and $^{\circ}\text{C}$ (Eqn. (9)) are connected with a single constant ('1.8') without an offset ('32'). A temperature difference in $^{\circ}\text{F}$, however is 1.8 times greater than the same difference in $^{\circ}\text{C}$. Thus, the Eqn. (9) can be considered as a basic conversion formula analogous to others (Table 1). Similarly, observing Eqn. (8) the temperature differences connecting $^{\circ}\text{C}$ and K units have no constants ($C_{\text{dif}} = K_{\text{dif}}$) involved, hence are same (e.g. $\text{J}/^{\circ}\text{C} = \text{J}/\text{K}$).

Example 5.10

What will be the heating rate of $5^{\circ}\text{F}/\text{min}$ in $^{\circ}\text{C}/\text{hour}$?

A $5^{\circ}\text{F}/\text{min}$ rate of heating means the "difference" in temperature is 5 units of $^{\circ}\text{F}/\text{min}$ (regardless of initial and final temperatures). Using Eqn. (9) and regular units conversion methodology we have:

$$5 \frac{\cancel{\text{F}}_{\text{diff}}}{\cancel{\text{min}}} \times \frac{1.8 \text{ C}_{\text{diff}}}{1 \cancel{\text{F}}_{\text{diff}}} \times \frac{60 \cancel{\text{min}}}{1 \text{ h}} = 540 \text{ }^{\circ}\text{C}/\text{h}$$

7 Other Resources

Several online resources and useful tools, such as conversion charts, tables, formulas, and web-based calculators or converters are available now-a-days to the users. For generality, units conversion tables, formulas, and charts usually provide conversion for an unit quantity and requires scaling for other or intermediate quantities; however, the web-based converters do automatic scaling. A basic search of the words "unit conversion" in Google will also brings in a web-based online calculator. Online calculators are convenient, quick, offer several optional units, and they do scaling calculations.

8 Summary

General users and especially students can be benefited from the understanding of the logic of the methodology of units conversion (factor-label or unit-factor) demonstrated. This methodology of carrying the units with numbers, as a ratio of unity, derived from the conversion formula, works well with most units. The method involves only basic multiplication, division, and canceling the units to finally arrive at the required results. All units with single constant conversion formulas of the style: $\text{Unit1} = \text{Constant} \times \text{Unit2}$; can be utilized in this method. The method is suited both for simple direct or advanced units conversions even that not available from other sources. Chain-conversion strategy demonstrated can be applied for advanced conversion involving compound units composed of several basic units. Square and cubic units can be derived from the regular units by squaring and cubing the quantities. One familiar limitation of the method is the inter-conversion of temperature units, namely, Fahrenheit, Celsius, and Kelvin. These units conversion formula are characterized by an offset and requires two constants; however, the temperature differences have no offsets. For quick units conversion, several online resources such as web-based converters or calculators and conversion charts or tables can be used.

References

Wikipedia. Units conversion by factor-label; 2004. http://en.wikipedia.org/wiki/Units_conversion_by_factor-label. Accessed on August 20, 2013.