Student Understanding of Vectors and Coordinate Systems Beyond Cartesian Coordinates in Upper Division Physics Courses

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Introduction

Understanding of Electricity & Magnetism in the upper-division requires an extensive integration of calculus concepts with abstract physics concepts. The ability to incorporate the use of vectors in several different coordinate systems is an essential skill in an E&M course since is used extensively. This study aims to understand how students think about coordinate systems and vectors in non-cartesian coordinate systems.

Research Questions

- What are the difficulties that student have when transitioning into using non-cartesian coordinate systems?
- What are the common misconceptions that students have about vectors and coordinates systems and how does that impact their learning of upper-division physics content, such as Electricity and Magnetism?

Methods

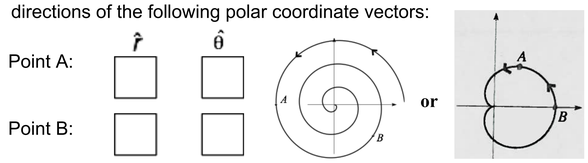
- This study probed students after all instruction in a math methods course for physics majors over three semesters
- In the course, students are taught mathematical techniques required for upper-division physics courses, including… (list everything you have down, make sure the spacing is correct.)
- For questions 1 and 2, the sample size was 28 students and for question 3 our sample size was 38 students

Results

Question 1

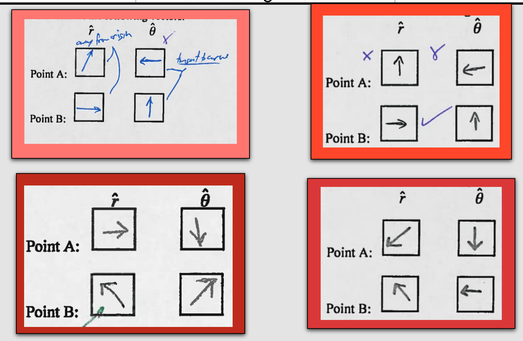
Consider the motion of a particle in a plane as shown, as it spirals inward toward the origin (circling the drain, perhaps?). For each motion, answer the question given.

For the locations show, draw an arrow in the box to indicate the directions of the following polar coordinate vectors:



CATEGORIES OF INCORRECT SOLUTIONS

ANSWER TYPE	DESCRIPTION	NUMBER OF STUDENTS
ø̂ Direction of Motion/Velocity	Were able to get all but 1 component correct (ø̂ for point A)	3
Off Axis	Off axis of heart path caused them to get point A incorrect	2
r̂ Direction of Motion	Thinking about r̂ are the direction of motion	2
Decreasing Direction	Having r̂ pointing in the decreasing direction	3



Question 2

Assume that the radial distance is $r=R_0-b\theta$ and the angle $\theta = \omega t$ where R_0, b, and ω are constants.
Indicate the units of R_0, b, and ω.

Determine expressions for the following in terms of in terms of the variables and constants given and the unit vectors $\hat{r}$ and $\hat{\theta}$.

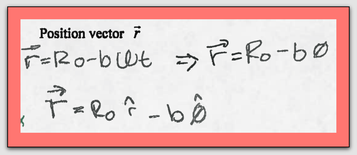
Position vector $\vec{r}$
Velocity vector $\vec{v}$

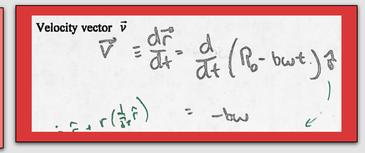
POSITION VECTOR: CATEGORIES OF INCORRECT SOLUTIONS

ANSWER TYPE	DESCRIPTION	NUMBER OF STUDENTS
Variables	Not in terms of the given variables	4
Vector	Not writing the complete vector	3
Tangential Component	Including a radial and a tangential component	3
Other	Responses did not seem to fit in the previous categories	3

VELOCITY VECTOR: CATEGORIES OF INCORRECT SOLUIONS

ANSWER TYPE	DESCRIPTION	NUMBER OF STUDENTS
Variables	Not in terms of given variables	2
Differentiating	Applying product rule but did not differentiate further	3
Product Rule	Did not apply product rule/ memorized the velocity vector	6
Other	Responses did not seem to fit in the previous categories	4





Question 3

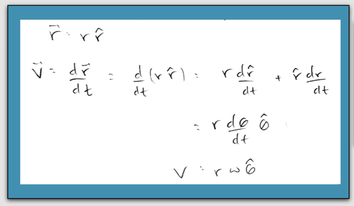
Start with the position vector $\vec{r}$ in plane polar coordinates. Determine the radial and tangential (theta) components of velocity in terms of r and θ and derivatives thereof.

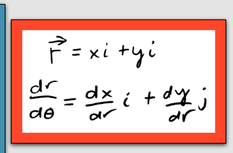
Velocity Vector: Incorrect solutions that began with $rr\hat{}$/ re_r / re_r/ or $r(\cos(ø)i\hat{}+\sin(ø)j\hat{}$

ANSWER TYPE	DESCRIPTION	NUMBER OF STUDENTS
Zero	Assuming (dr/dt) is zero	4
Differentiating	Did not differentiate $r\hat{}$/ or e_r	1
Variables	Forgetting variables/symbols or not in terms of specified variables	3
Rules	Did not apply Chain rule/ or Product rule	3

Velocity Vector: Incorrect solutions that began with incorrect position vectors

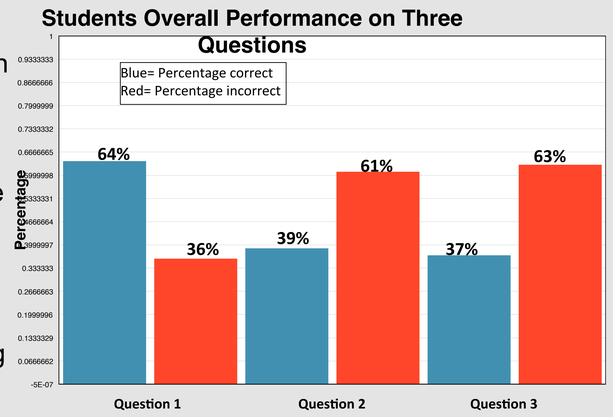
ANSWER TYPE	DESCRIPTION	NUMBER OF STUDENTS
Pattern Matching	Thinking x is equivalent to r and y is equivalent to theta	1
Tangential Component	Thinking the position vector has both a radial and tangential component	4
Confusion with Differentiating	Simply not knowing to differentiate with respect to time	5
Other	Did not fit into any of the previous categories	3





Discussion

- Unit vectors in coordinate systems other than Cartesian coordinates, i.e., polar coordinate systems, seem to present a difficulty for students in the upper division
- Students appear to be more competent when using vectors and coordinate systems depending on the context, students struggle when required to relate their mathematics understanding to physical problems
- Some students appear to be over reliant on Cartesian coordinate systems and demonstrate difficulties when interpreting position and velocity vectors in non-cartesian coordinate systems



Future Work

- This work raises questions about how students understand vectors in spherical coordinates and what difficulties they may have that might impact their ability to learn physics
- We have generated questions that will attempt to probe student understanding of spherical coordinates that we plan to use to collect both student written and interview data

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