Beyond the exam score: Gauging conceptual understanding from final exams in Calculus II

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MOTIVATIONS

- Instructors want their students to have a "conceptual understanding" of the topics in their mathematics courses, but they do not always know how to find evidence for this.
- Researchers in Undergraduate Math Education have found evidence in in-depth interviews with students, classroom observations, and student responses on tasks created *specifically* for the purposes of an educational study.
- However, an instructor is typically limited to homework, exams, and interactions in the classroom or during office hours.

In this research, we gauge the extent to which one can bridge the gap between an instructor's desire to evaluate conceptual understanding and the limited information he/she often has for this evaluation.

RESEARCH QUESTIONS

Using student responses to final exam questions from a Calculus II course, we want to know:

1. How can we use information about the problem-solving behaviors that students demonstrate when solving final exam problems to make inferences about their degrees of conceptual understanding?

2. How can we use the answers to Question 1 to create new instructional tools that would provide more evidence for degrees of conceptual understanding?

METHODS: CODE CREATION

Lit Review

Our research was most influenced by Dubinsky & McDonald (2001), Mejia-Ramos et al. (2011), Thurston (1994), and Krathwohl (2002)

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Course Review

- "Calculus: Early Transcendentals" (Rogawski, 2011)
- final exam, test, and group homework questions

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Initial Frameworks

- Levels of understanding guided by APOS
- Problem-solving behaviors guided by Mejia-Ramos et al. (2011), Thurston (1994), Krathwohl (2002)

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Coding/Revision Round 1

- Coded all students, specific questions only
- Revision goal: create better fit for behaviors seen in round 1

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Coding/Revision Round 2

- Coded 10 from all 4 sections, all questions
- Revision goal: identify general problem-solving behaviors, combine codes, use revised Bloom's taxonomy vocab

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Coding Round 3/ Finalization

- Subset of 24 from round 2 for final individual results, overall results from revised round 2 coding
- Final codes ordered from Krathwohl (2002) general hierarchy

FINAL CODES AND EXAMPLES

Higher ↑ Conceptual understanding ↓ Lower

Analyze Relationships (AR): student compares math objects to justify a claim

Generate Examples (GE): student gives example to refute or support a claim

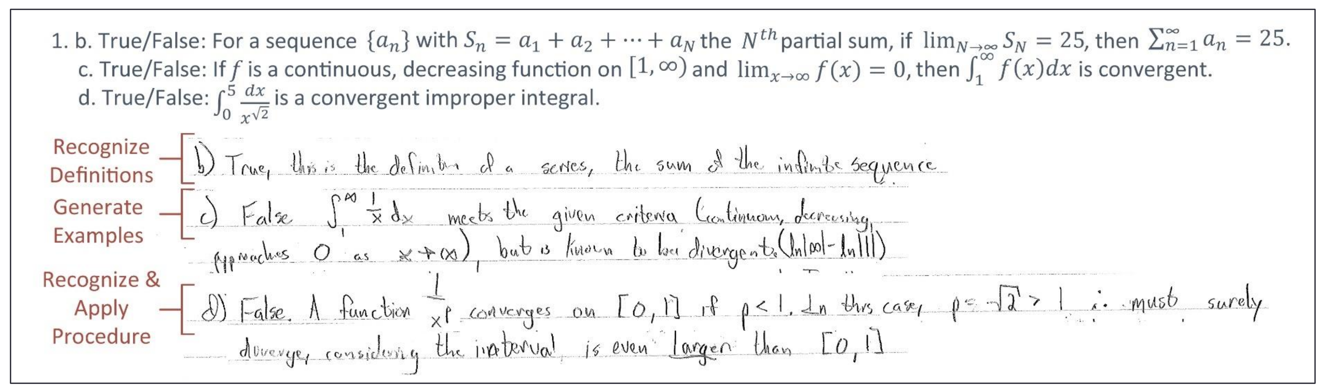
Recognize Details (RD): student uses a detail or feature of a math concept that is not stated in the problem within the problem-solving process

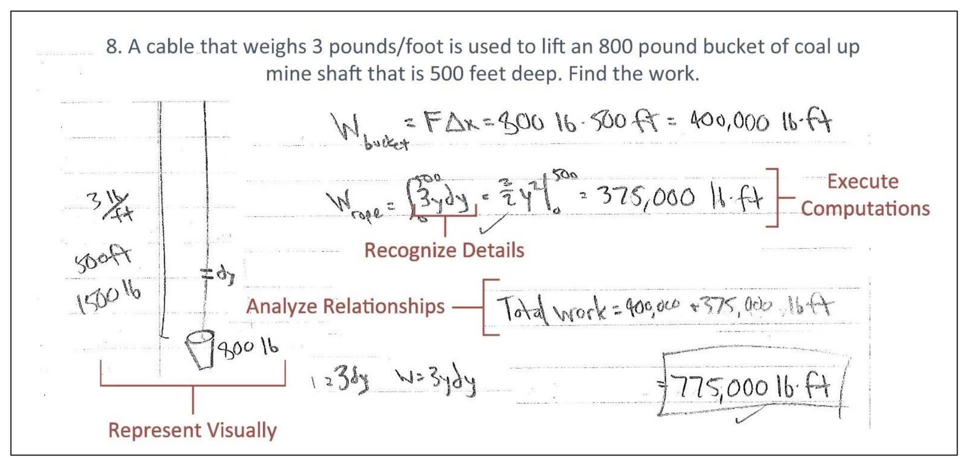
Recognize and Apply Procedures (RAP): student cites or applies a theorem, test, or formula from Calculus

Represent Visually (RV): student provides a visual aid

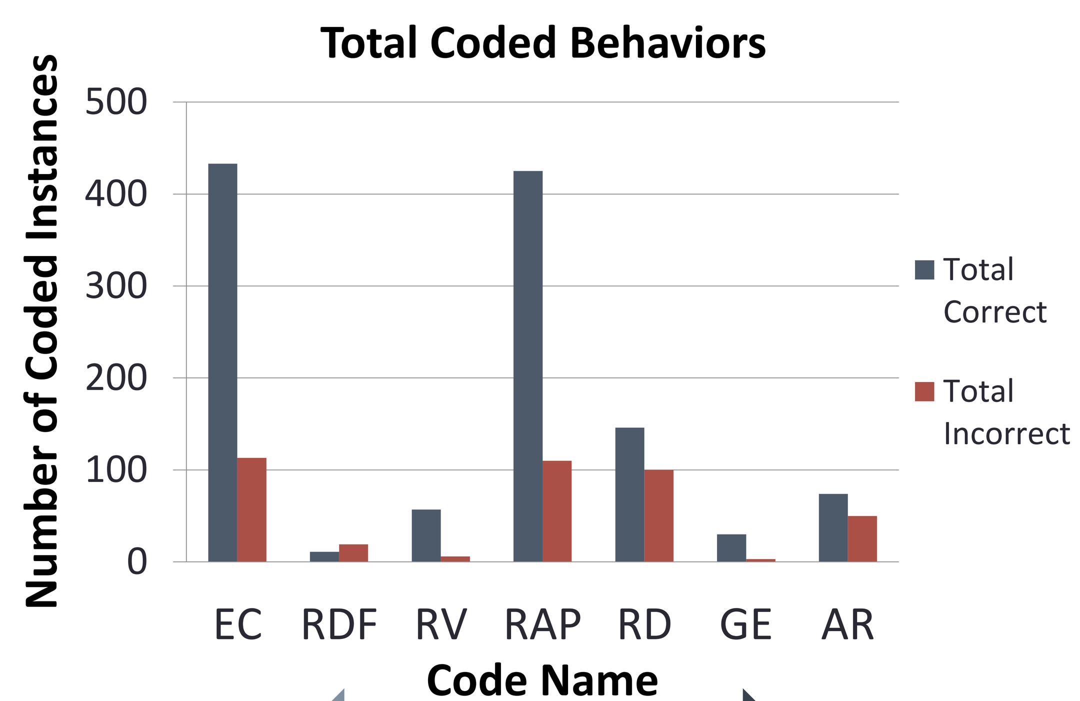
Recognize Definitions (RD): student describes or cites a definition

*It should be noted that our final codes only represent the spectrum from low to medium conceptual understanding within the hierarchy of Bloom's Taxonomy; there exist higher-level behaviors beyond AR that were not present in our data set.





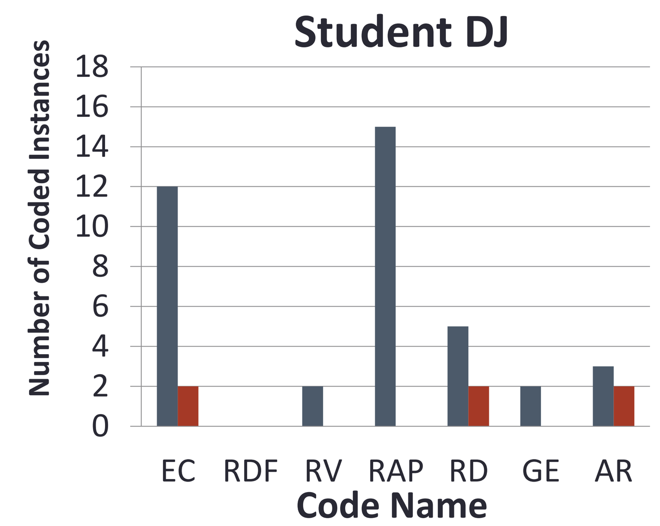
COURSE RESULTS



Features:

- Clusters in **computations (EC)** and **procedures (RAP)**, both require relatively low conceptual understanding, very low proportional error
- Large error in **details (RD)** and **relationships (AR),** both require relatively higher conceptual understanding, may indicate lack of stress on context

INDIVIDUAL RESULTS

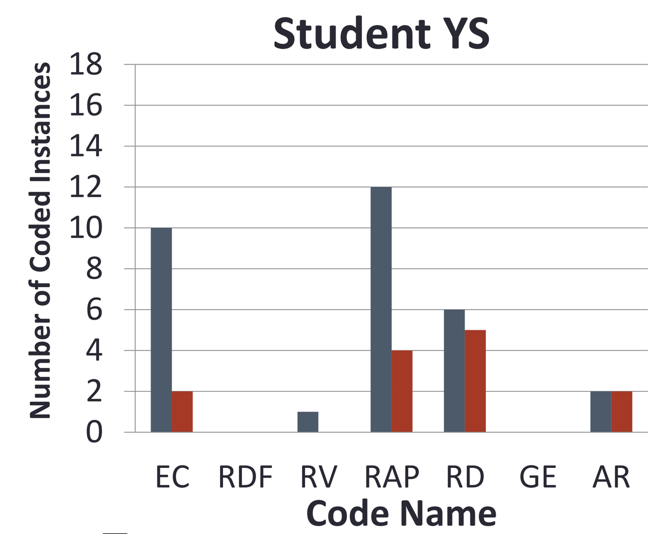


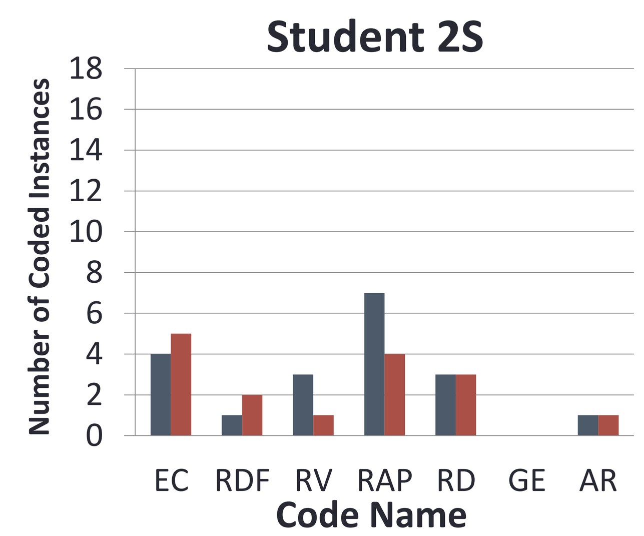
Conceptual understanding →

- Low proportion of error throughout, may see slightly more proportional error in higher level behaviors than low-level
- All or almost all behaviors displayed
- No extreme irregularities from shape of overall exam graph

Conceptual understanding →

- Low error proportions in low-level behaviors, high error proportions in higher-level behaviors
- Some behaviors may be missing or very infrequent
- Shape of graph closely resembles that of the overall exam graph





Conceptual understanding →

- Almost all behaviors have high error proportion, especially low-level
- Graph clusters more to the left than overall exam graph
- Higher-level behaviors are less frequent and/or have a very high error proportion

CONCLUSIONS

- **Success on the exam was largely determined by performance on low-level tasks (EC, RAP).** Students most often struggled with areas of higher understanding (RD, AR) based on error proportions and frequency.
- It's likely that students with high levels of understanding either did not need to demonstrate higher-level behaviors or did not have the opportunity. Thus, it was difficult to differentiate between students that memorized how to do a problem from class and those that genuinely considered the problem context. **We were more confident categorizing students with lower levels of understanding, but the lack of justification in student work made differentiating between medium and higher levels difficult.**
- We often coded only one or two behaviors per question, but demonstrating knowledge in multiple ways can be indicative of higher-level understanding. **In future exams, explicitly designing problems that require one or more types of justification of work, or prompt students to display behaviors typical of each level of conceptual understanding would make coding more accurate.**

REFERENCES

- Dubinsky, E., & McDonald, M. A. (2001). APOS: A constructivist theory of learning in undergraduate mathematics education research. *The teaching and learning of mathematics at university level* (pp. 275-282). Springer Netherlands.
- Krathwohl, D. R. (2002). A revision of Bloom's taxonomy: An overview. *Theory into practice*, *41*(4), 212-218.
- Mejia-Ramos, J. P., Fuller, E., Weber, K., Rhoads, K., & Samkoff, A. (2012). An assessment model for proof comprehension in undergraduate mathematics. *Educational Studies in Mathematics*, *79*(1), 3-18.
- Thurston, W. P. (1998). On proof and progress in mathematics. *New directions in the philosophy of mathematics*, 337-55.

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