Understanding of Non-Cartesian Coordinates: Resource Activation in Cylindrical and Spherical Coordinates

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Introduction:

As part of a larger effort to develop a more precise mathematical curriculum for upper division physics, we studied physicists at different level's thought processes on different coordinate systems. Prior research posits that students have significant difficulty working both numerically and visually with non-Cartesian coordinate systems, especially the notion that the spatial directions of variables changes depending on time and position. We report findings from one-on-one interviews designed to facilitate subject thought in this area.

Research Questions:

1. How does upper division physics students' understanding of coordinate system influence their ability to conceptualize and solve physics problems?
2. How does the understanding upper-division physics students have of cylindrical coordinates compare to their understanding of spherical coordinates?

Methods:

- Developed questions in physical and non-physical contexts involving unit vectors based on previously identified student difficulties
- Think-aloud interviews conducted where students solved questions on a board while being asked additional follow-up questions to further explore reasoning
- Recruited 3 volunteers for interviews: one upper-level physics undergraduate, one 1st year graduate physics student, and one tenured physics professor
- Interviews were video recorded and lasted 45-70 minutes

Theoretical Framework:

We analyzed these interviews from a Resources Framework (Hammer et al). In the work of Sayre & Wittman, resources are described as **small reusable pieces of thought** that make up concepts and arguments. Resources are **unconscious cognitive units** that students "activate" when framing a problem or concept in a certain way. The way a student frames a problem affects what resources might be activated.

References

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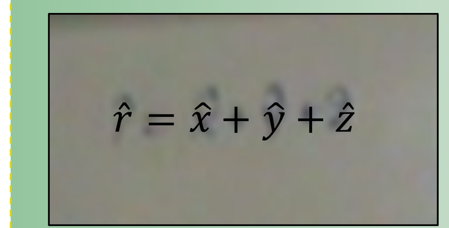
Student thinking in cylindrical and spherical coordinates:

Activation of Unit Vector Resources

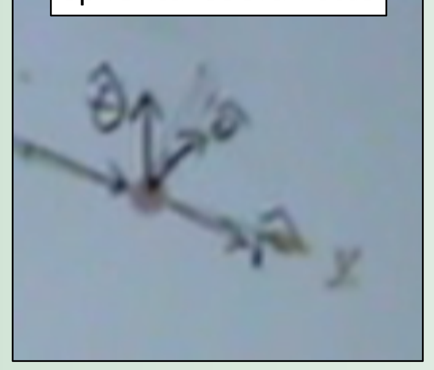


How David (undergraduate) defined the position vector. This shows that he has a non-static $\hat{r}$ resource, but it is not yet fully developed.

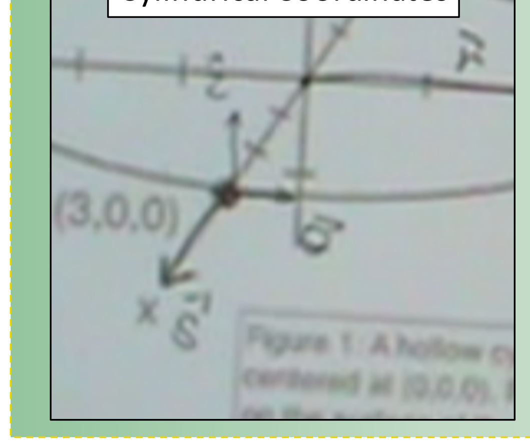
When asked to draw unit vector directions

How Kris (graduate) drew the spherical unit vectors, noting that they must create an orthogonal system

Spherical Coordinates



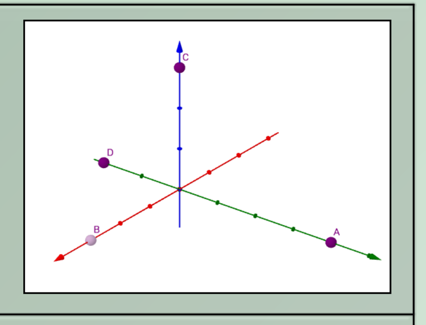
Cylindrical Coordinates



"You can in principle take this point [points at C] and just move it to any of the other points and keep the relative orientation to the three unit vectors. And then orient this triad in such a way that the *s* vector points outward, and the *z* vector points upward." -Steve (Physics Faculty)

Conflation of Cartesian Resources

Question 3: For each of the points *A, B, C,* and *D* in Figure 3, write the position vector $\vec{r}$ in terms of the unit vectors $\hat{r}, \hat{\theta}, \hat{\phi}$.
Correct Answer at Point A: $\vec{r} = 4\hat{r}$



David's Answer*:	$\vec{r} = 2\hat{r} + \hat{\phi} + \hat{\theta}$
Kris's Answer:	$\vec{r} = r_A\hat{\phi}$
Steve's Answer:	$\vec{r} = 4\hat{r} + \frac{\pi}{2}\hat{\phi}$

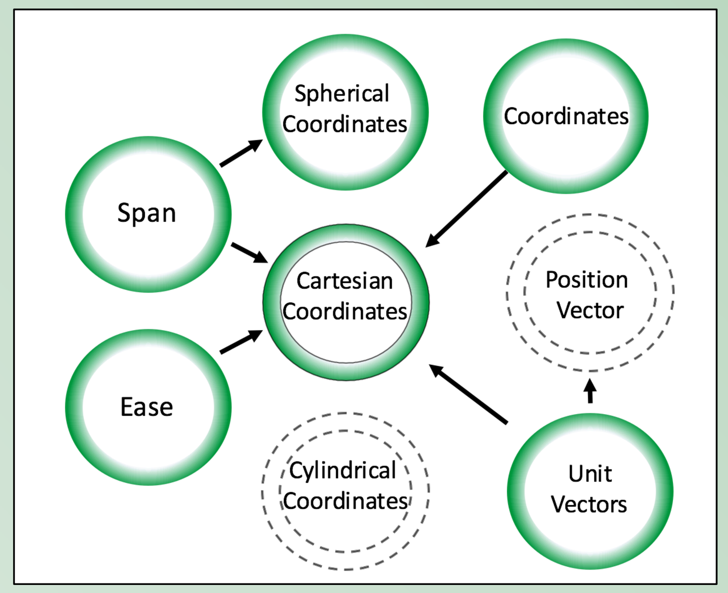
→ Each subject answered nearly the same in question 1, which was a similar question, but in cylindrical coordinates

*David mentions that he is unsure of the magnitude of the $\hat{\theta}$ and $\hat{\phi}$ vectors and that they might be zero.

As explained by Hinrichs, the most likely explanation for these responses is "pattern-matching", i.e. mapping coordinate values onto unit vectors, which works only in Cartesian coordinates. This is further reinforced by the work of Farlow (2016). During the Interview with Kris, he repeatedly said that the angle must be "swept out" from the x-axis, which is a conflation of coordinate resources to vector component resources and possible evidence of improper transfer from Cartesian coordinates.

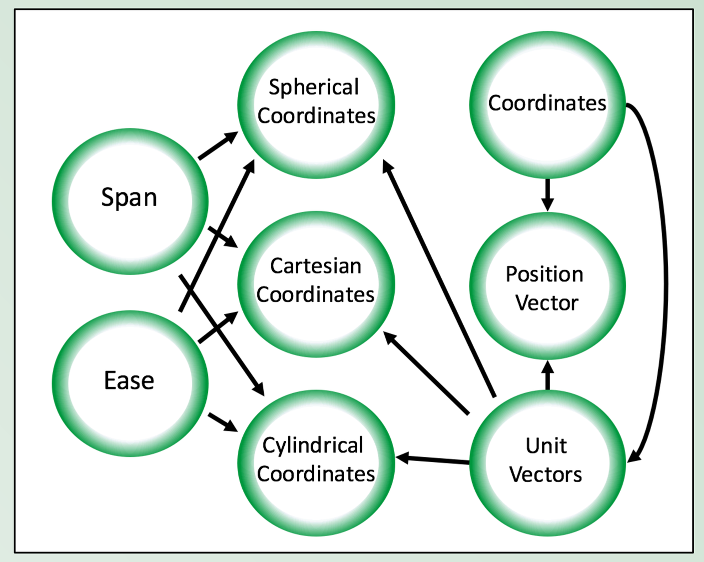
Works:	$(x, y, z) = x\,\hat{x} + y\,\hat{y} + z\,\hat{z}$
Do not work:	$(s, \theta, z) \neq s\,\hat{s} + \theta\,\hat{\theta} + z\,\hat{z}$ $(r, \theta, \phi) \neq r\,\hat{r} + \theta\,\hat{\theta} + \phi\,\hat{\phi}$

Connections Between Potential Resources



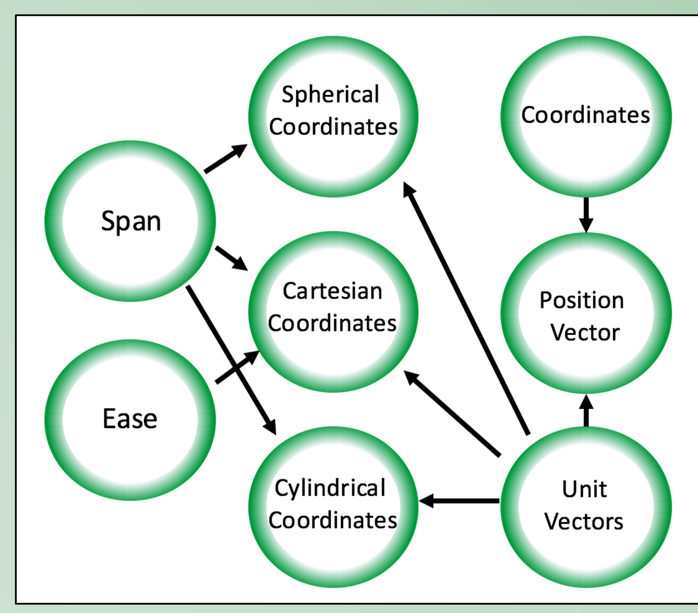
David (Undergraduate student)

- Still building connections between resources
- Some resources not yet fully developed



Kris (Graduate student)

- More durable resources
- Connections are reinforced but not yet concrete



Steve (Physics Faculty)

- Resources are robust throughout interview
- Connections between resources are explicit

Conclusions:

- Interview subjects had a tendency to utilize Cartesian resources when solving problems in non-Cartesian coordinates
- Subjects' resources for Cartesian coordinates were more solid and easily accessible than non-Cartesian coordinate resources
- Subjects transfer Cartesian resources to cylindrical coordinates as has been documented previously with spherical coordinates

Future Work:

- Further development of math and physics problems that emphasize use of non-Cartesian unit vectors and quantity vectors
- Further explore the use of a Resources and Framing Theoretical Frameworks to our data
- Develop instructional materials to encourage appropriate resource activation