

## MOTIVATIONS

- Instructors want their students to have a “conceptual understanding” of the topics in their mathematics courses, but they do not always know how to find evidence for this.
- Researchers in Undergraduate Math Education have found evidence in in-depth interviews with students, classroom observations, and student responses on tasks created *specifically* for the purposes of an educational study.
- However, an instructor is typically limited to homework, exams, and interactions in the classroom or during office hours.

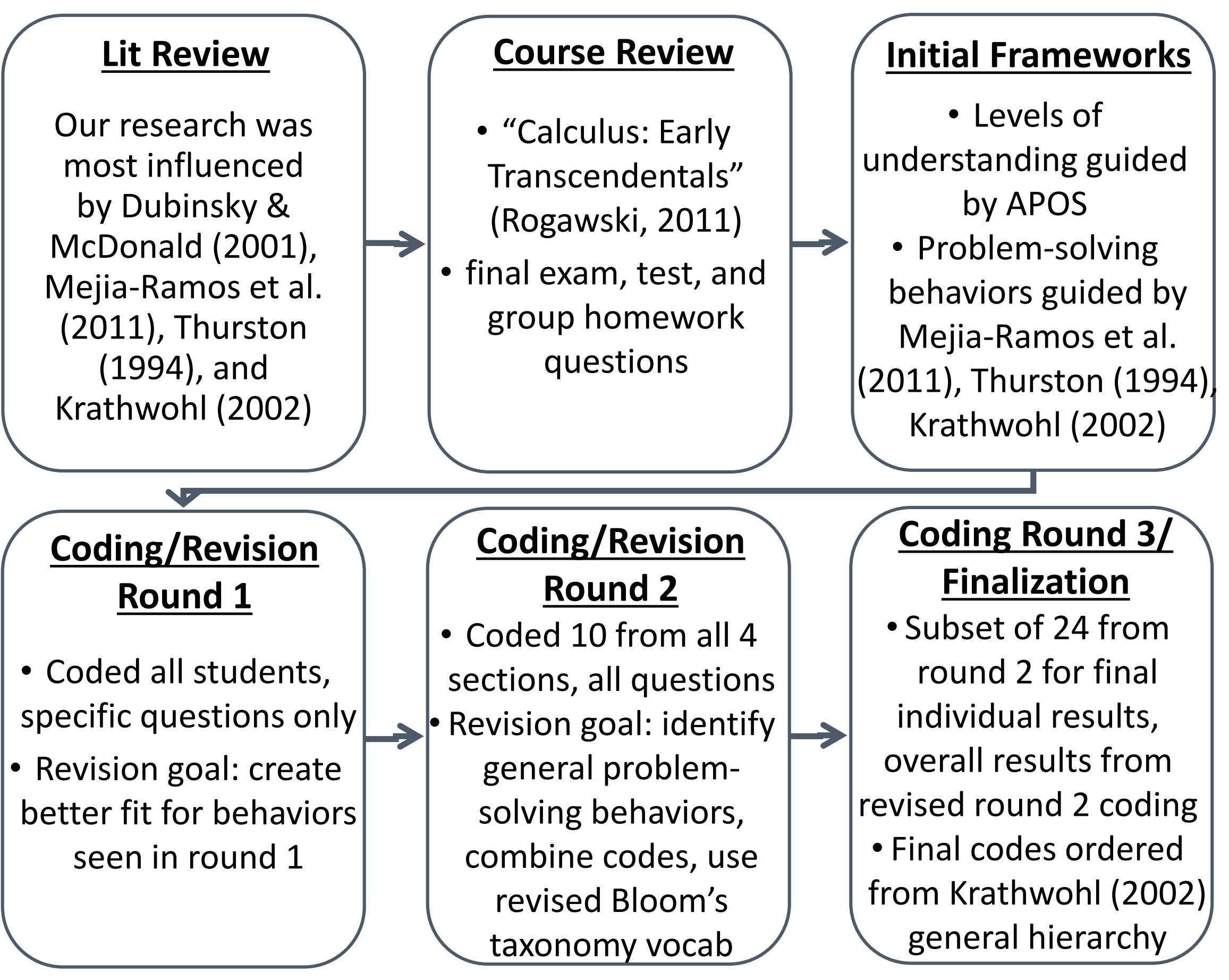
In this research, we gauge the extent to which one can bridge the gap between an instructor's desire to evaluate conceptual understanding and the limited information he/she often has for this evaluation.

## RESEARCH QUESTIONS

Using student responses to final exam questions from a Calculus II course, we want to know:

- How can we use information about the problem-solving behaviors that students demonstrate when solving final exam problems to make inferences about their degrees of conceptual understanding?
- How can we use the answers to Question 1 to create new instructional tools that would provide more evidence for degrees of conceptual understanding?

## METHODS: CODE CREATION



## FINAL CODES AND EXAMPLES

- Higher  
↑ Conceptual understanding
- Analyze Relationships (AR):** student compares math objects to justify a claim
  - Generate Examples (GE):** student gives example to refute or support a claim
  - Recognize Details (RD):** student uses a detail or feature of a math concept that is not stated in the problem within the problem-solving process
  - Recognize and Apply Procedures (RAP):** student cites or applies a theorem, test, or formula from Calculus
  - Represent Visually (RV):** student provides a visual aid
  - Recognize Definitions (RD):** student describes or cites a definition
- Lower

\*It should be noted that our final codes only represent the spectrum from low to medium conceptual understanding within the hierarchy of Bloom's Taxonomy; there exist higher-level behaviors beyond AR that were not present in our data set.

1. b. True/False: For a sequence  $\{a_n\}$  with  $S_N = a_1 + a_2 + \dots + a_N$  the  $N^{\text{th}}$  partial sum, if  $\lim_{N \rightarrow \infty} S_N = 25$ , then  $\sum_{n=1}^{\infty} a_n = 25$ .  
 c. True/False: If  $f$  is a continuous, decreasing function on  $[1, \infty)$  and  $\lim_{x \rightarrow \infty} f(x) = 0$ , then  $\int_1^{\infty} f(x) dx$  is convergent.  
 d. True/False:  $\int_0^{\infty} \frac{dx}{x^2}$  is a convergent improper integral.

Recognize Definitions: [D] True, this is the definition of a series, the sum of the infinite sequence.  
 Generate Examples: [C] False.  $\int_0^{\infty} \frac{dx}{x^2}$  does not meet the given criteria (continuous, decreasing, approaches 0 as  $x \rightarrow \infty$ ), but it is known to be divergent (p-test).  
 Recognize & Apply Procedure: [D] False. A function  $f$  converges on  $[0, 1]$  if  $p < 1$ , in this case  $p = 2 > 1$ , it must surely diverge, considering the interval is even larger than  $[0, 1]$ .

8. A cable that weighs 3 pounds/foot is used to lift an 800 pound bucket of coal up mine shaft that is 500 feet deep. Find the work.

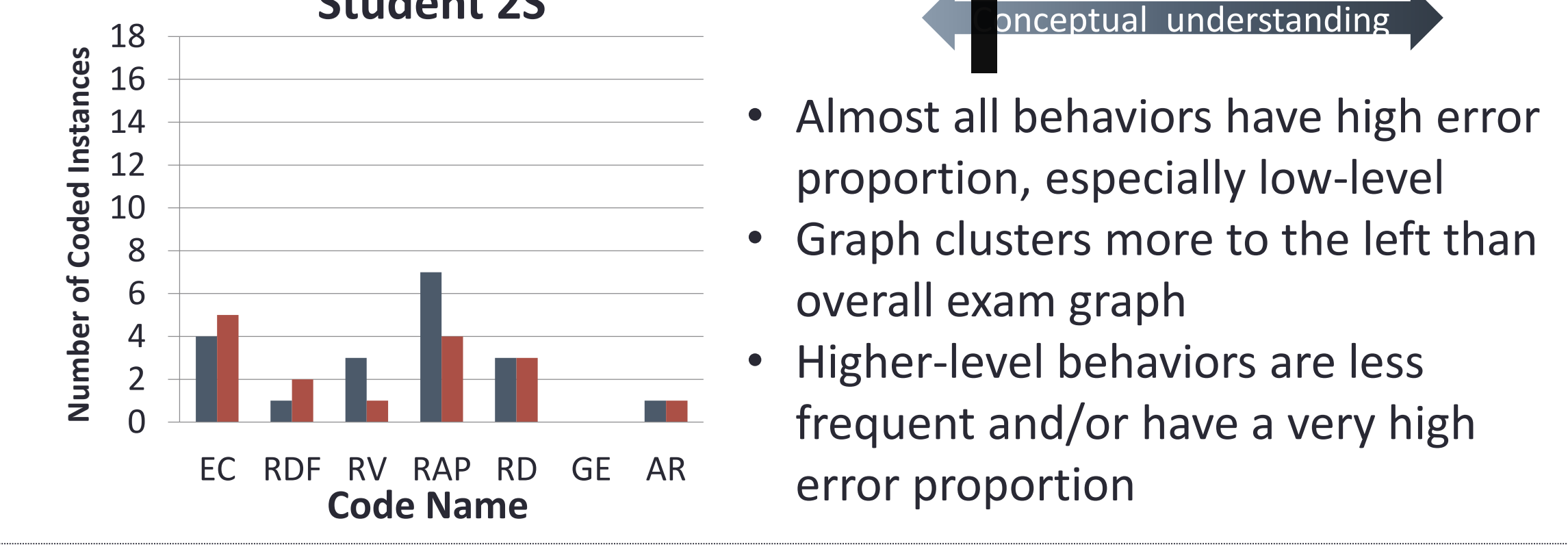
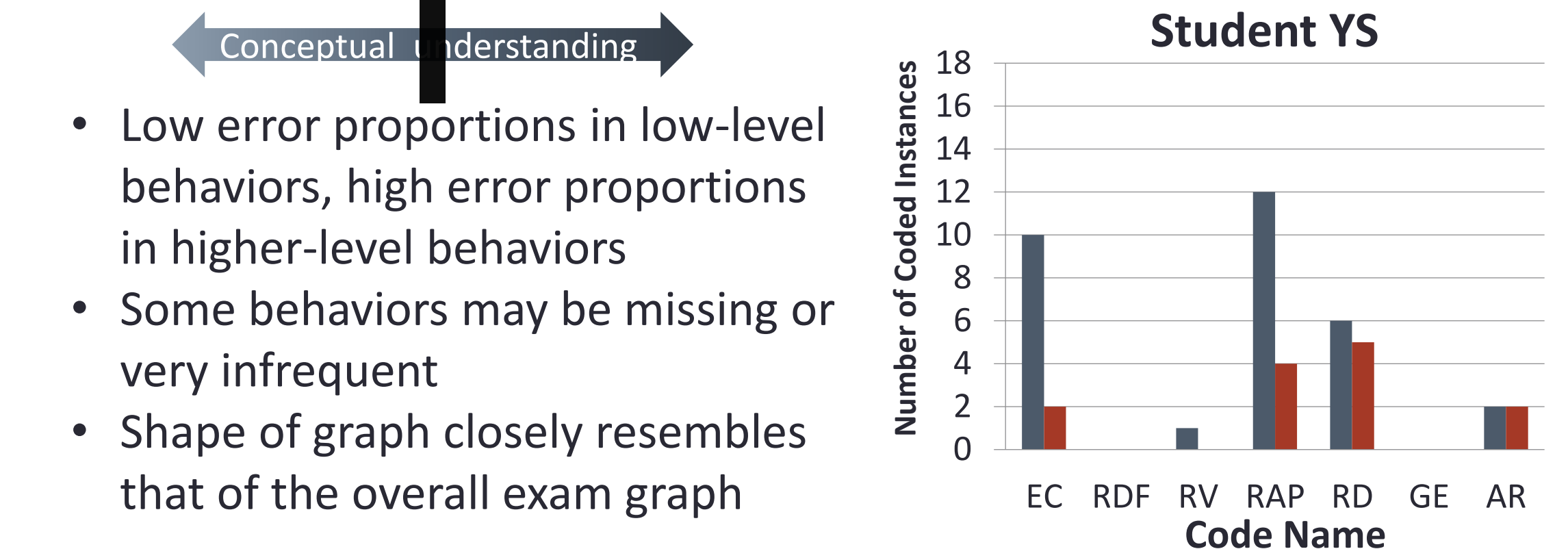
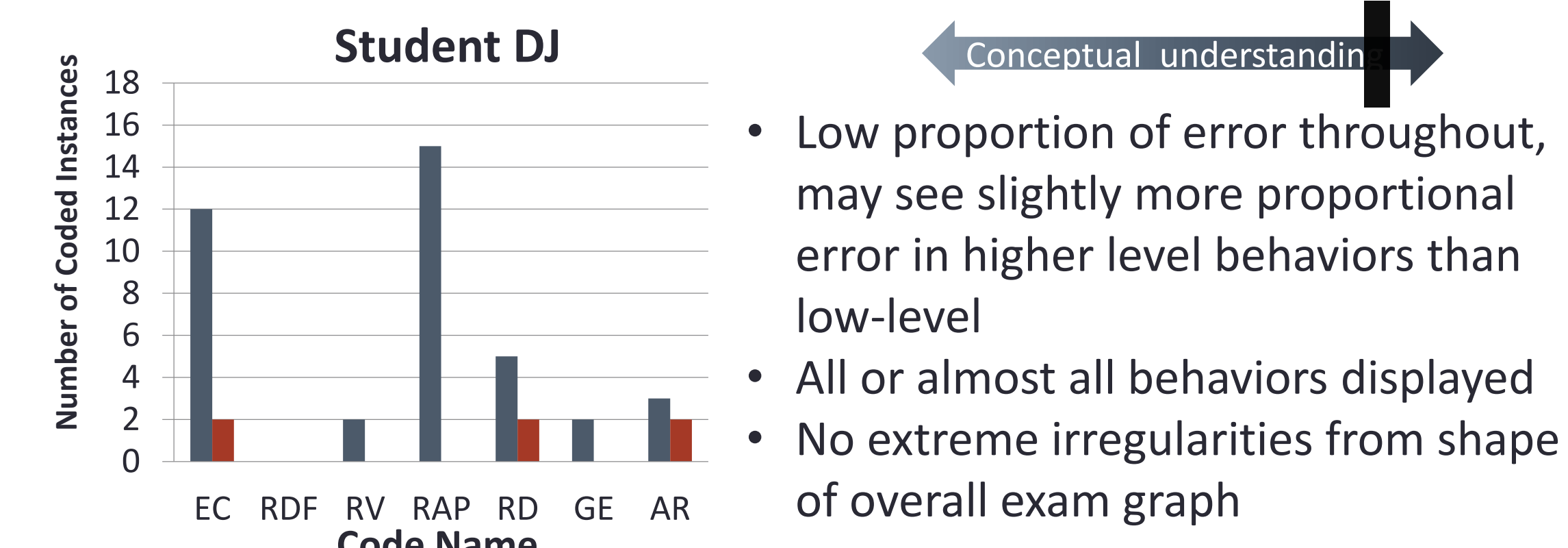
$W_{\text{bucket}} = F \Delta x = 800 \text{ lb} \cdot 500 \text{ ft} = 400,000 \text{ lb}\cdot\text{ft}$

$W_{\text{cable}} = \int_0^{500} 3y \, dy = \frac{3}{2} y^2 \Big|_0^{500} = 375,000 \text{ lb}\cdot\text{ft}$  Execute Computations

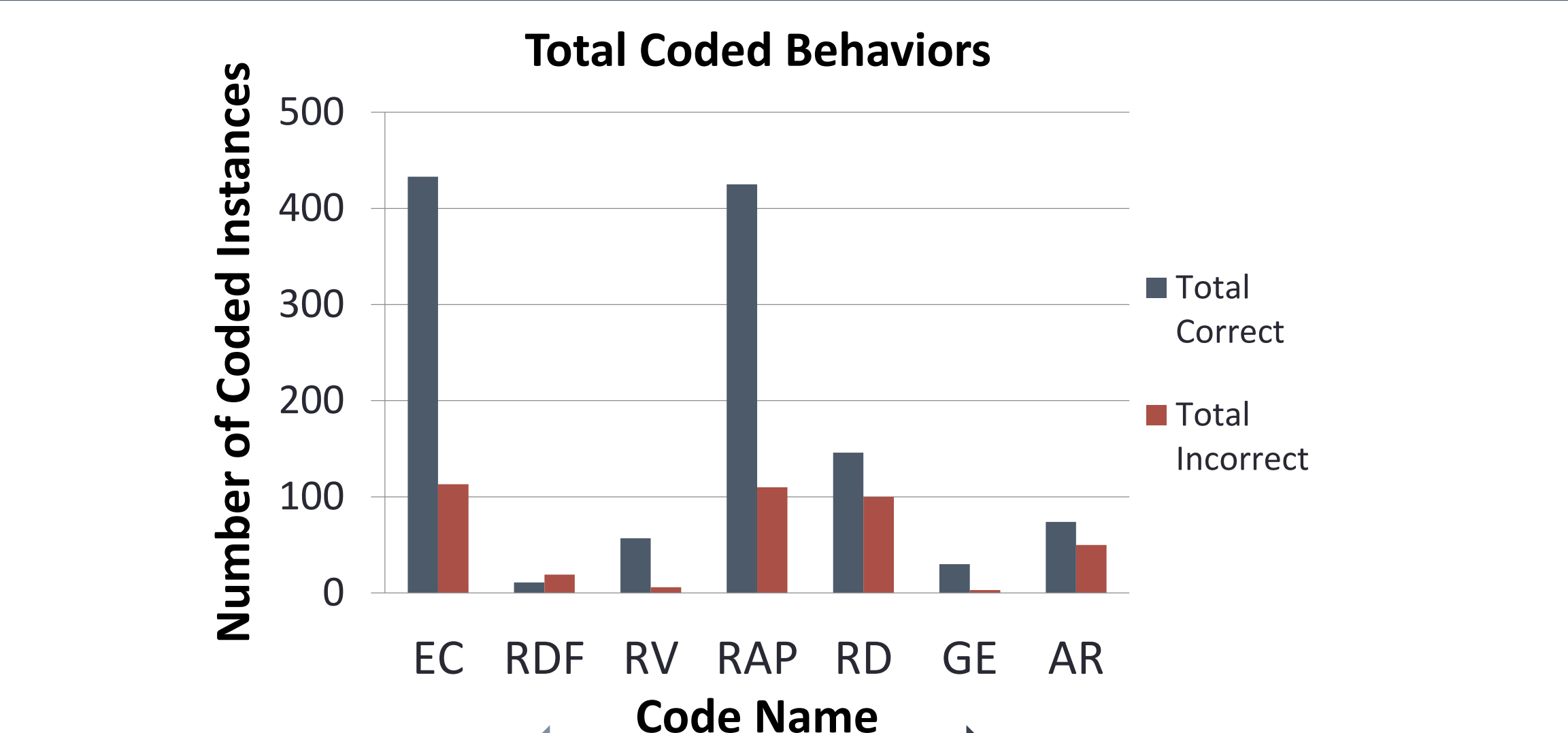
Recognize Details: Analyze Relationships: Total work =  $400,000 + 375,000 \text{ lb}\cdot\text{ft}$

Represent Visually:  $775,000 \text{ lb}\cdot\text{ft}$

## INDIVIDUAL RESULTS



## COURSE RESULTS



- Features:**
- Clusters in **computations (EC)** and **procedures (RAP)**, both require relatively low conceptual understanding, very low proportional error
  - Large error in **details (RD)** and **relationships (AR)**, both require relatively higher conceptual understanding, may indicate lack of stress on context

## CONCLUSIONS

- Success on the exam was largely determined by performance on low-level tasks (EC, RAP).** Students most often struggled with areas of higher understanding (RD, AR) based on error proportions and frequency.
- It's likely that students with high levels of understanding either did not need to demonstrate higher-level behaviors or did not have the opportunity. Thus, it was difficult to differentiate between students that memorized how to do a problem from class and those that genuinely considered the problem context. **We were more confident categorizing students with lower levels of understanding, but the lack of justification in student work made differentiating between medium and higher levels difficult.**
- We often coded only one or two behaviors per question, but demonstrating knowledge in multiple ways can be indicative of higher-level understanding. **In future exams, explicitly designing problems that require one or more types of justification of work, or prompt students to display behaviors typical of each level of conceptual understanding would make coding more accurate.**

## REFERENCES

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