Multivariable Calculus Textbook Analysis Highlights a Lack of Representation for Non-Cartesian Coordinate Systems

Chaelee Dalton,[1] Brian Farlow,[2] and Warren Christensen[2]

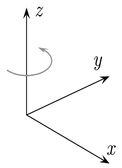
[1]Pomona College, [2]North Dakota State University

Motivation

Upper-division physics courses (Electricity and Magnetism, Classical Mechanics, Quantum Mechanics, etc.) employ mathematical knowledge in non-Cartesian coordinate systems. To understand what mathematical knowledge students have gained with respect to these systems, we turn to the math class these systems are taught extensively in, Multivariable Calculus. This work will inform both physics and mathematics professors on students' mathematical preparedness, specifically with non-Cartesian coordinate systems, when entering upper-division physics courses.

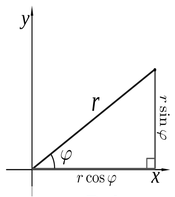
Mathematical Background

Coordinate systems use ordered pairs or triples of numbers, known as coordinates, to describe unique points, shapes, or surfaces in space, either 2D or 3D. Traditional mathematics focuses on Cartesian coordinate systems, but other systems are advantageous in mathematical and physics applications.



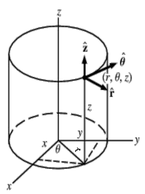
Cartesian (Rectangular)

Point: (x,y) or (x,y,z)
Applications: any linear motion



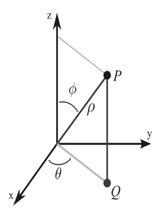
Polar

Point: (r,θ)
Applications: pendula, 2D radial motion, orbits



Cylindrical

Point: (r,θ,z)
Applications: cylindrical capacitor, electrical field of line charge



Spherical

Point: (ρ,θ,ϕ)
Applications: radial 3D fields, Maxwell speed distribution, Schrodinger's equation

Methods

This study surveys seven popular Multivariable Calculus textbooks, and one Mathematics Methods in the Physical Sciences textbook. The study analyzes these textbooks both qualitatively and quantitatively.

Qualitative

Some structural elements focused on included

- The chapters non-Cartesian coordinate systems appear in
- The topics in those chapters where non-Cartesian coordinates are used
- Separation between topics in non-Cartesian coordinates and Cartesian coordinates

Quantitative

The items deemed most important were

- Definitions, properties, and theorems
- Examples
- Problems and exercises

They were coded by coordinate system as shown below.

Sample Coding Exercise

instructions: spherical

33–34 ■ Evaluate the integral by changing to spherical coordinates.

cues: spherical **notation: cartesian**

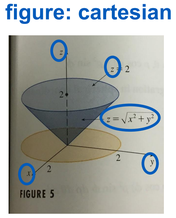
$$\int_{-a}^{a}\int_{-\sqrt{a^2-y^2}}^{\sqrt{a^2-y^2}}\int_{-\sqrt{a^2-x^2-y^2}}^{\sqrt{a^2-x^2-y^2}} (x^2z + y^2z + z^3)\,dz\,dx\,dy$$

(Stewart, 2016, 1051)

Coding Criteria

- Which system's notation is used?
- Which system's instructions/language is used?
- Which system do any accompanying figures favor?
- Which system is used in the solutions for examples?
- Which coordinate system could be "cued" by a problem?

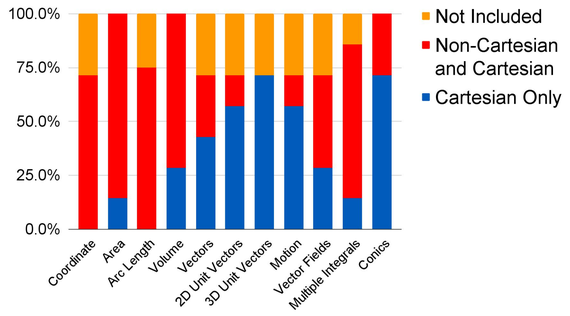
figure: cartesian



(Stewart, 2016, 1043)

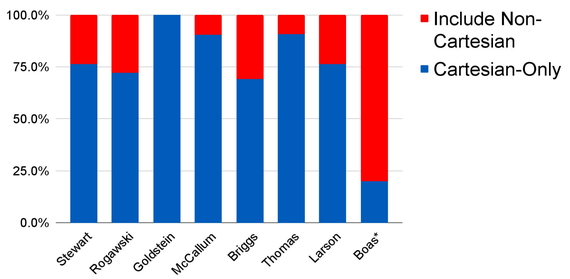
Results

Inclusion of Non-Cartesian Coordinate Systems Across Various Topics in Multivariable Calculus Books



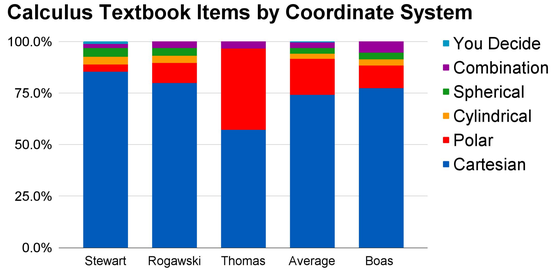
Topics are always introduced in Cartesian coordinates. Topics in non-Cartesian coordinates and Cartesian coordinates are separated by chapter subsection.

Percentage of Chapters of Books Which Include Non-Cartesian Coordinate Systems



Only 18.7% of chapters in Calculus textbooks included any instance of non-Cartesian coordinates.

Calculus Textbook Items by Coordinate System



Those chapters which did include non-Cartesian coordinates still focused predominantly on Cartesian coordinates (74.0%).

Discussion

Mostly Cartesian Content

Both quantitative and qualitative results suggest that Cartesian coordinate systems are heavily favored over non-Cartesian coordinate systems, over many different topics in Multivariable Calculus Textbooks.

Surface-Level Non-Cartesian Content

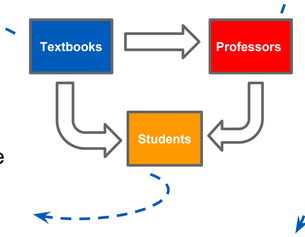
Moreover, the content that does include non-Cartesian coordinate systems is mostly surface level. This both includes the types of topic covered and the types of problems asked.

- Involvement of non-Cartesian coordinate systems consists of "converting" from one coordinate system to another
- Many topics fail to include non-Cartesian coordinate systems altogether

Future Work

The textbook study was conducted because the textbook informs professors on the material and students as a resource and for problems.

A student survey will be developed, implemented, and analyzed to directly understand student knowledge of non-Cartesian coordinate systems.



Several Multivariable Calculus professors, who have a direct impact on student learning, will be interviewed.

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