Investigating the Prevalence of Non-Cartesian Coordinate Systems in Upper-division Physics Textbooks

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Motivation

Students in upper-division physics struggle to use non-Cartesian unit vectors[1] even after completing a math course designed specifically for physics majors, called Math Methods. About Math Methods:

- It's a course within undergraduate physics curriculum
- It does **not** exist/is not required at every university
- It's typically taken after students complete lower- and middle-division math courses **(fig. 2)**
- It's intended to prepare students for upper-division physics coursework

To understand why students struggle with non-Cartesian coordinate systems, a textbook investigation was initiated in the course where students are first introduced to non-Cartesian coordinate systems: Multivariable Calculus. Textbooks were examined because they inform course instructors and serve as a practical resource for students.

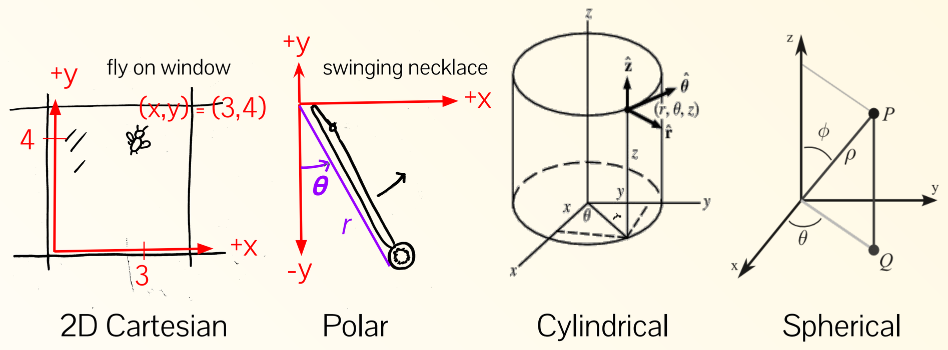


Figure 1: Coordinate Systems as "Mathematical Framings"

The analysis of seven commonly used calculus textbooks revealed:[2]

- **18.7%** of textbook **chapters** included **any instance** of **non-Cartesian coordinates**
- Within the chapters containing non-Cartesian coordinates, **74.0%** of examined **content** focused predominantly on **Cartesian** coordinates
- **non-Cartesian unit vectors** were **completely absent**

To investigate if non-Cartesian coordinates are prevalent in math methods and upper-division physics courses, an analogous textbook study was initiated for a math methods textbook and two-upper division physics textbooks.

Methods

Three textbooks were chosen for initial analysis: 1) Boas's *Mathematical Methods in the Physical Sciences* 2) Thorton & Marion's *Classical Dynamics* and 3) Griffiths's *Introduction to Electrodynamics*. Within each book, content within was identified as one of the following:

- Instructional content (definitions, theorems, and properties)
- Example problems
- Independent exercises

For each item in the categories above, three aspects were examined to identify a coordinate system(s): notation, explicit instructions, and associated figures. For example problems, the solution was examined; and for independent exercises, the presence of "cues" were sought.

The original coding questions (Fig. 3) were adapted for use on the math methods textbook (Boas, chapter 2) and two upper-division textbooks: an analytic mechanics textbook (Thornton & Marion, 5.1 and 5.2) and an electricity & magnetism textbook (Griffiths, 2.1). This required re-interpreting the coding questions to capture the format and features of the physics texts that did not exist in the calculus texts.

A sample of items for each textbook is shown below. The items presented for the math methods and physics books were chosen because they exhibit characteristic(s) that resulted in re-interpreting the coding questions. The calculus book (Rogawski) is shown as a baseline from which the original coding questions were developed.

Q1: Does the **notation** of the item imply a particular coordinate system?

Q2: Does the item **explicitly state** to use or report an answer in a particular coordinate system?

Q3: Does the **accompanying figure** favor a particular coordinate system?

Q4: (examples only): Does the **solution** use a particular coordinate system to solve?

Q5: (exercises only): Are there **cues** in the item that suggest solving using a different coordinate system than what Q1, Q2, or Q3 suggests?

Figure 3: Coding Questions

(Q1) Notation is Cylindrical.
Definitions and theorems are clearly marked.

(Q5) $x^2+y^2=\square^2$ and *disk* are "polar cue words" in calculus books.

(Q4) Solution is Cylindrical.
Solution presents steps explicitly.

(Q3) Figure is Cartesian

(Q1) Notation is Cartesian.

(Q2) *Explicit* instructions are Polar.

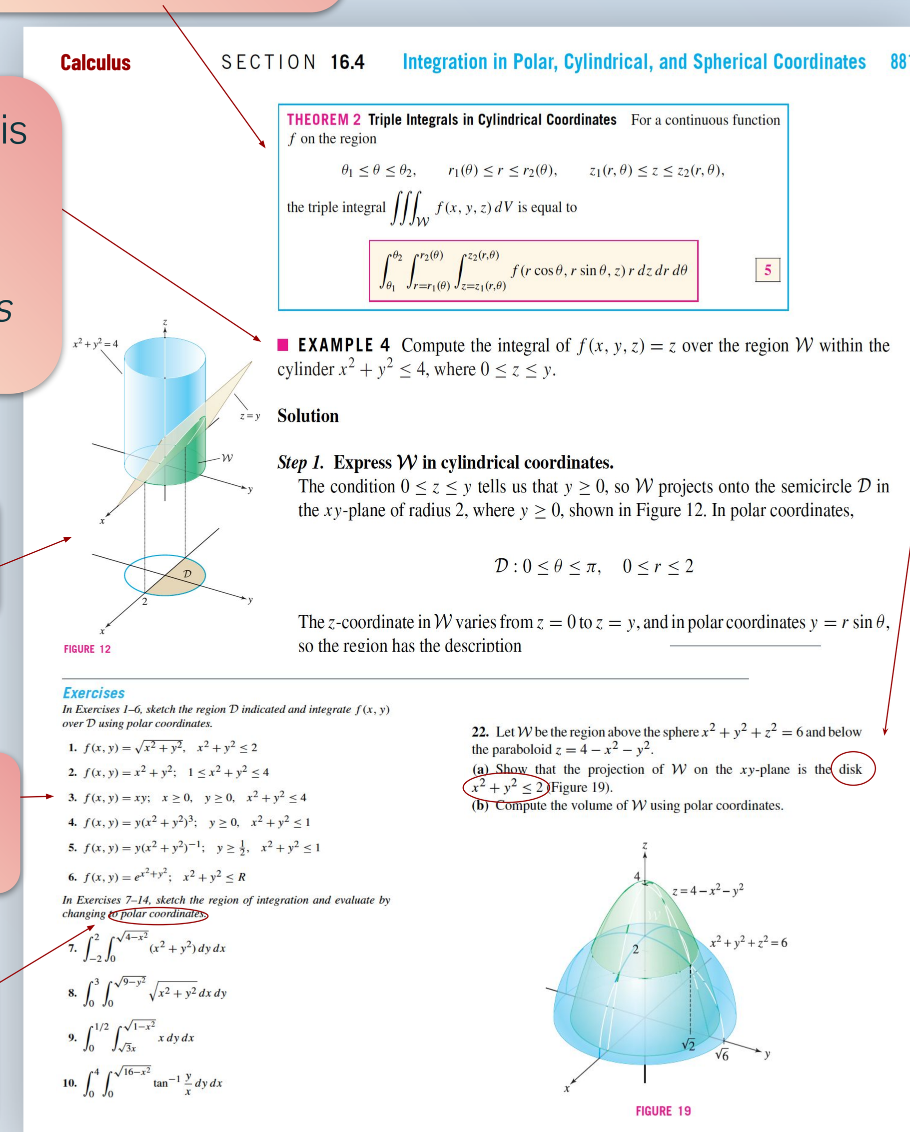


Fig. 2.4a needed to interpret original problem in Griffiths.
Notation may include figures. Figs. coded for both Q1 and Q3.

𝓇 ("script r") is not equal to r from polar coordinates.

Fig. 2.4b coded as Cartesian because θ not "polar" in this context.

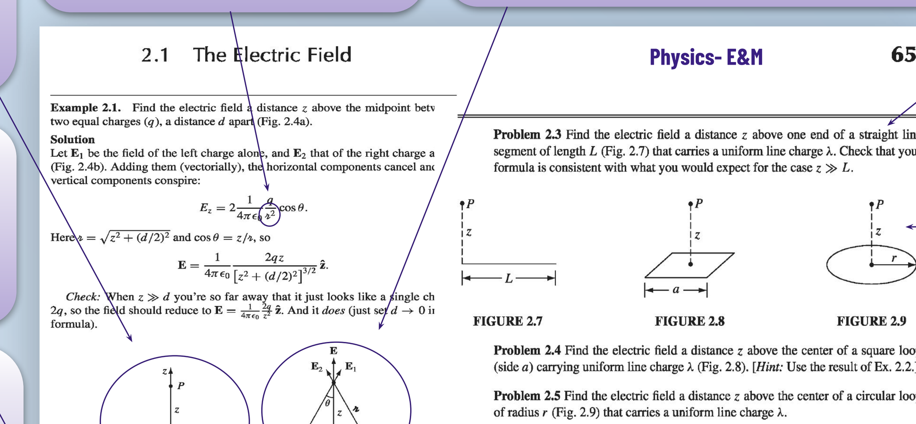
(Q1) What constitutes "notation" for physics texts?

(Q5) Fig. 2.9 can be cueing cartesian, polar, cylindrical, or spherical coordinates. Coded as You Decide.

Fundamental physical law not boxed.
Indicates need to operationalize def ns for instructional content.

(Q1) "r" denotes a linear distance here ≠ not polar coordinate "r". Coded as Dummy.

(Q1, Q3) Figures without axes is common (fig. 2.7-2.9)

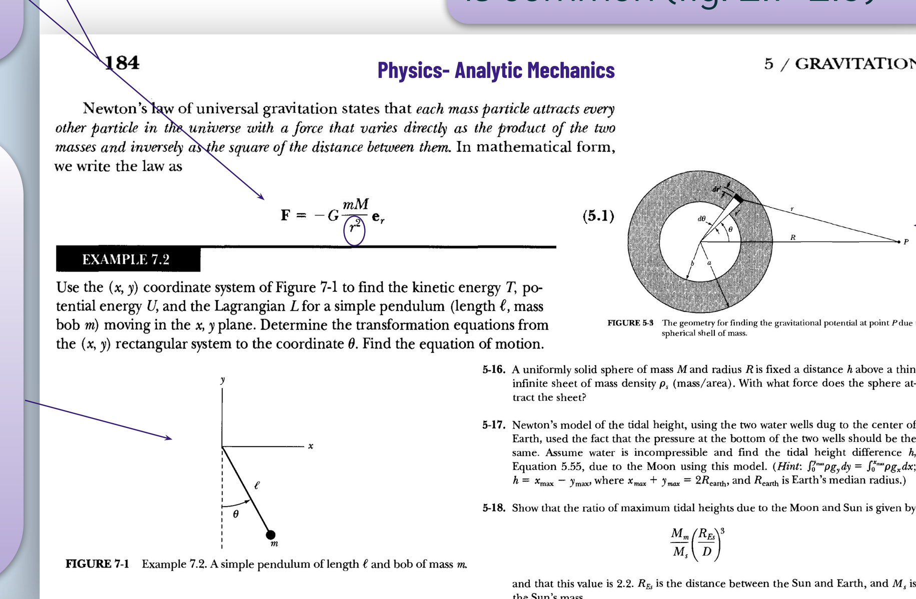


(Q2) No explicit instructions to report answer in any coordinate system; final result in Cartesian form.
Exhibits normalization of Cartesian coordinates

(Q1) Radial distance denoted as "ℓ" instead of "r".
Coordinate system variables given physical meaning in physics books.

(Q3) Figure is Spherical.
Figures often shown in cross-section. Coder must read caption.

Exercises rarely provide figures in T&M book.



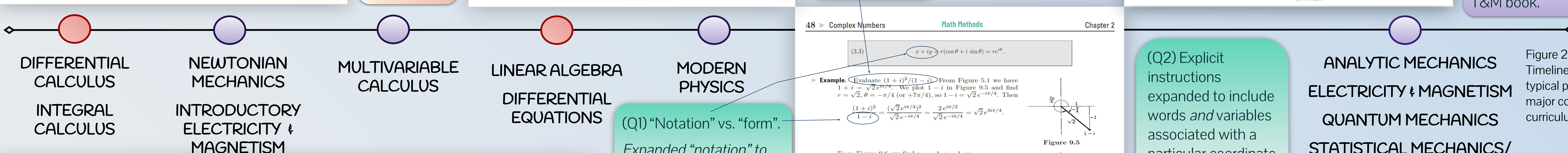
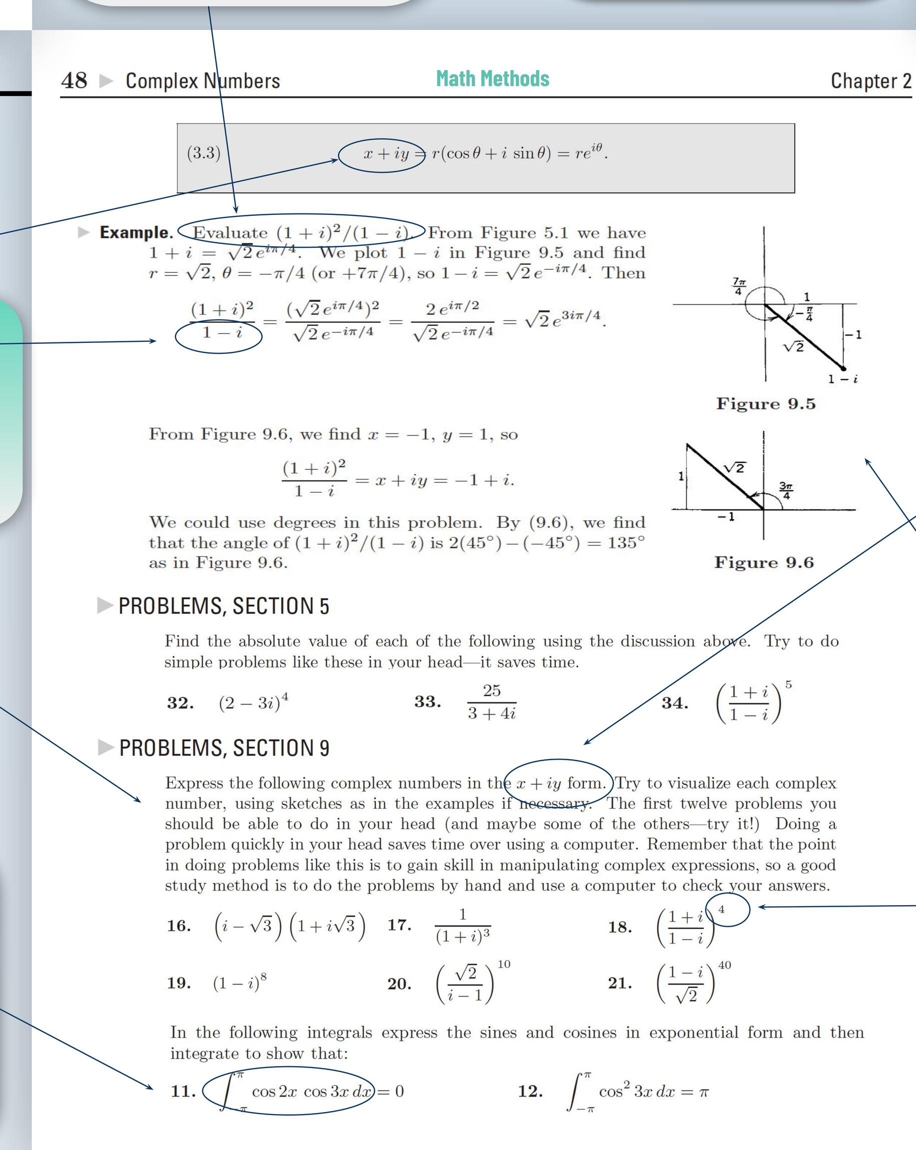


Figure 2: Timeline of a typical physics major core curriculum.

(Q1) "Notation" vs. "form".
Expanded "notation" to include form for Q1.

(Q2) Long instructions for exercises appear; requires closer interpretation.

(Q1) Notation appears to be cartesian but x is a *placeholder* for an angular input θ. Coded as Dummy.



(Q2) Explicit instructions expanded to include words *and* variables associated with a particular coordinate system.

(Q3) Both figures include cartesian and polar coordinates; coded as Mixed.

(Q5) Power '4' or higher represents cue to convert to polar coord. in #18 but power '4' not a cue in #32.
Exhibits coding is context-dependent in Math Methods book.

Implications & Future Directions

Coding one chapter of a math methods book and three sections between two, upper-division books required deeper examination and frequently re-interpreting the coding for Q1, Q2, Q3, and Q5. This revealed salient differences across all three books:

- The math methods book contains features reminiscent of both the calculus and physics books
- In format and content, the physics books differ substantially from the calculus books
- Awareness of the difference between the math presentation in calculus vs. physics books can help upper-division physics instructors tailor their courses to inclusively transition students

To obtain a more comprehensive understanding of the prevalence of non-Cartesian coordinates in the math methods and upper-division physics books, the uncoded sections/chapters will be coded using the same method. Coding more items may require further re-interpretation of the coding questions.

References

[1]Vega et. al (2016) [2]Dalton et. al. (2018) [3]Rogawski, J., Adams, C. (2015) *Calculus* [4]Boas, M.L. (2005) *Mathematical Methods in the Physical Sciences* [5]Griffiths, D.J. (2017) *Introduction to Electrodynamics* [6]Thorton, S.T., Marion, J.B. (2003) *Classical Dynamics of Particles and Systems.*

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