Applying the Resource-based Ontology to Undergraduate Quantum Mechanics

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Collaborations in Discipline-based Education Research (CiDER)

What factors affect students' ability to transfer knowledge across the subjects of Linear Algebra and Quantum Mechanics? And how can that transfer be measured?

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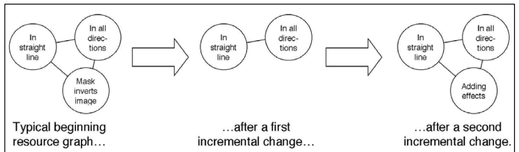
Theoretical Framework

Resources

The resource-based ontology of cognitive structure posits that students compile their explanations in real time from conceptual resources that are neither right nor wrong.[1]

There are two major types of resources: conceptual and epistemological. Conceptual resources deal with understanding physical phenomena, such as one's understanding of the concept of force and Newton's Laws. Epistemological resources deal with monitoring the conceptual resources that are being activated, such as using facts and formulas versus reasoning and sense-making.[1]

The figure below shows resource graphs used to represent linked ideas when reasoning about specific contexts in physics.[3] The resource graphs shown here are undergoing incremental change in learning about light and shadow. Two incremental changes take place, one deletion and one addition.



Incremental change of resources graphs in studying light and shadow.[3]

Framing

The resources that the student activates in any particular situation depend on how they frame the problem they are considering,[1] that is, how they answer the questions, "What is going on here?" and, "How should I approach knowledge?" The former question addresses conceptual resources to be activated, and the latter addresses the epistemological resource.

Frames are locally coherent sets of resource activations.[1] The process of learning involves forming these sets of activations, and then, once formed, using these frames in settings where they seem applicable. Our investigation aims to identify factors which may cause students to change frames, and thus access (or not access) certain locally coherent sets of resources, in the course of compiling an explanation.

References

[1] Hammer, David, et al. "Resources, framing, and transfer." *Transfer of learning from a modern multidisciplinary perspective (2005): 89-120.*

[2] Henderson, Frances, et al. "Symbol sense in linear algebra: A start towards eigentheory." *Proceedings of the 13th Special Interest Group of the Mathematical Association of America on Research in Undergraduate Mathematics Education Conference on Research in Undergraduate Mathematics Education. 2010.*

[3] Wittmann, Michael C. "Using resource graphs to represent conceptual change." *Physical Review Special Topics-Physics Education Research 2.2 (2006): 020105.*

The Interviews

Students were recruited from a first semester course in quantum mechanics to participate in a semi-structured problem solving interview, conducted by an NDSU physics faculty member. The interview protocol was taken directly from a Mathematics Education Research study,[2] which investigated students' ideas about matrix multiplication in linear algebra. The protocol contained 3 mathematical expressions that the students were asked to interpret, along with a number of follow-up questions. The interviews were videotaped and subsequently transcribed.

Interview Sections

The interviews were broken down into smaller sections, each of which is between 1 and 3 minutes. Cues in the students' responses and behavior indicated when sections should be broken. These cues include:

- the student pausing when they were confused or had "run out of steam" in an explanation
- the student concluding an explanation of a written calculation
- the student explicitly stating that their explanation had concluded
- the interviewer asking a new question from the protocol when he saw fit to do so.

Emergent Keywords

Emergent terminology in the students' explanations (called "keywords") were tallied by interview section, and put into a table (below). The keywords have been grouped by their association with the subject of mathematics, physics, or both. These words did not occur anywhere in the language of the interview protocol. They emerged from the student's choice of words as they compiled their explanations, responding to the mathematical expressions in the protocol.

Interview Protocol

1. "Different people read expressions in math differently. I'm curious to know how you read this expression." $\begin{bmatrix} 1 & 3 \\ 6 & -1 \end{bmatrix}\begin{bmatrix} 2 \\ 3 \end{bmatrix}$

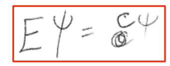
2. "Consider a 2x2 matrix A and a vector $\begin{bmatrix} x \\ y \end{bmatrix}$. How do you think about $A\begin{bmatrix} x \\ y \end{bmatrix} = 2\begin{bmatrix} x \\ y \end{bmatrix}$?"

3. "Suppose $A = \begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}$. Now how do you think about $A\begin{bmatrix} x \\ y \end{bmatrix} = 2\begin{bmatrix} x \\ y \end{bmatrix}$?"

Subject	Keyword	Ben's Interview by Sections																			
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Math	Matrix	X		X		X		X	X	X	X	X									
	Matrix Multiplication	X	X	X	X	X		X			X										
	System of Equations		X								X										
	Linear Algebra								X									X			
Neutral	Equivalence		X					X		X	X		X								
	Solutions	X	X	X						X	X	X	X	X	X	X				X	X
	Written Calculation			X							X				X						
	Eigenvalue							X								X	X			X	X
Physics	Quantum Mechanics							X								X	X	X		X	X
	Operator							X								X	X			X	X
	Wavefunction							X									X			X	X
	Eigenstate															X	X				

Data & Results

"Well the first thing that pops in my head is a little bit of quantum, where I think, we have: 'A' times a matrix equals a constant times a matrix, which makes me think of the eigenvalue expression we've been using in quantum mechanics, where we have an operator acting on a wavefunction (writing) always returns an eigenvalue of the wavefunction, times the wavefunction." (Sections 7)



Claim: The student's language indicates a change in frame from the first expression given in the protocol to the second. In sections 1 through 6 there are no keywords from QM present in the student's explanation, but when the student is given the second expression the student identifies it as the "eigenvalue expression" from QM, and continues by explaining the expression with keywords that are commonly seen in QM.

"When I see the equals sign, I guess it tells me everything on the left half is the same as everything on the right... So I assume that A has to be 2." (Section 9)

After being given the explicit values for A, in the third protocol question:

"I said 'hmm, makes me think A was equal to the constant,' and now we have a matrix and a constant and, hmm, doesn't quite sound equal... I know what I can do is I can try to multiply some of this through and see what it looks like." (Section 10)

Claim: The student sees the discrepancy between his initial conclusion that A=2 and the explicitly given matrix, A. The student decides to "multiply some of this through" rather than activating conceptual resources to reconcile his despite not knowing of the form of the solution that he may attain. Note, the student uses linear algebra keywords ***exclusively*** during this line of explanation.

"One of the things we learned in quantum ...when you measure the state of a system you're not going to get an eigenstate, you're going to get an eigenvalue; because a system, when measured, is going to be an eigenvalue of its eigenstate." (Section 15)

"An object exists in an eigenstate, and we apply--(pauses)--(confidently) we apply this operator and basically it returns the eigenvalue." (Section 16)

Claim: The student's repeated activation of the concept of eigenvalues in the context of QM has lead to a structurally stable set of activations, and the establishment of the eigenvalue/QM frame as a cognitive object itself.[1] The concept of the eigenvalue is structurally tied in with the student's understanding of QM.

Future Work

We will continue to analyze additional interview data, and the emergent keywords that students use in explaining their conceptual understandings. We are also developing complimentary methods for incorporating information about epistemological resources into our analysis.

	Keywords	1*	2	3	4*	5	6	7*	8	9	10	11	12	13	14	15	16	17	18	19
Math	Linear Algebra							x				x		x						x
	Math											x								
	Matrix multiplication	x	x																	
	Multiply (or "times")	x		x	x		x	x					x							
	Row	x		x			x													
	Equal (or "equivalent")	x			x	x	x	x			x		x							
	Column vector	x																		
	Matrix				x								x		x					
	Column						x													
	Divide							x												
	Integers									x										
	Real numbers									x										
	Complex numbers									x										
	Matrix system										x									
	Linear combination										x									
	Linear system										x									
	Powers of 'x'										x									
	Characteristic equation												x							
	Lambda												x							
	Identity Matrix												x							
	Function													x				x		
	Vector													x						
Eigen-terms	Eigenvector equation				x															
	Eigenfunction				x						x			x	x	x	x			
	Eigenvector													x		x	x			
	Eigenvalue													x		x	x			
Physics	Quantum mechanincs											x							x	x
	Wavefunction (or "Psi")											x					x	x		
	Operator																x	x		
	Schrodinger equation																	x		
	Hamiltonian																	x		
	Potential energy																	x		
	Energy																	x	x	
	Allowed energy																	x	x	
	Quantum system																		x	
	Quantized																		x	