Identifying Resources for Integration Across Math & Physics

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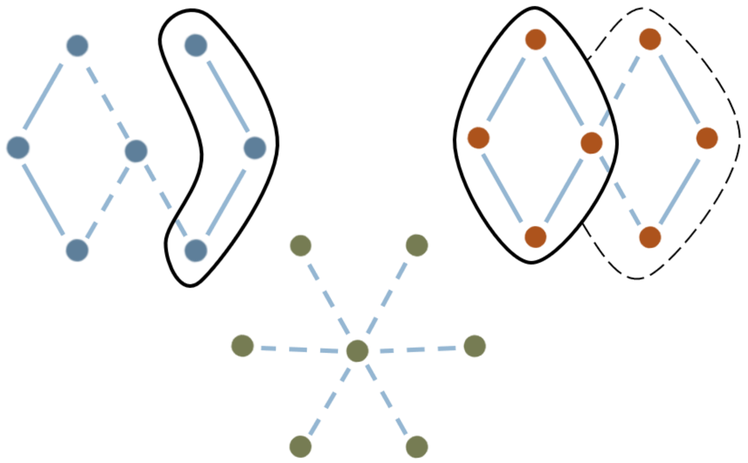
Resources and Framing



Resource diagrams illustrating how students may be subconsciously connecting ideas.

Interview Protocol[1]

Math I: 
Two wires are attached to two telephone poles. Suppose we wanted to know the area between the two wires. How could you figure that out?

Math II: $\int_1^2 \frac{2}{x^3} - x^2\, dx$
Compute and then discuss this integral.

Math III: $\int \sin(x) \quad \int_2^0 e^x\, dx$
Look at these integrals and talk about what they mean.

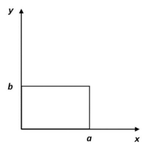
Math IV: $\int dx \quad \int \sqrt{t}\, dx$
Look at these integrals and talk about what they mean

Math V: $-2\int_D f(x)dx$
Suppose we have a function with domain D. What does this integral mean?

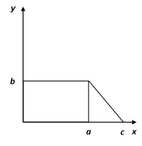
Math VI: 
This picture shows the outline of a violin body. If you wanted to know the area of this shape, how could you figure that out?

Phys I: $\int \frac{nRT}{V} dV$
n, *R*, & *T* are constants. Tell me how you think about this expression?

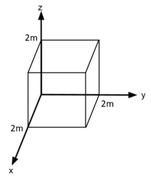
Phys II: $\int dm$
In this expression m is mass. Can you tell me how you think about this expression? What might this expression be equal to, if anything?

Phys III: 

Can you use calculus to find the area of this box?
Can you talk about all the important elements of what you wrote down?
Can you write down an integral that would give you an area?

Phys IV: 

Can you use calculus to find the area of this shape?
With regard to shape III. What does the expression $\int_0^b a\, dy$ mean to you?

Phys V: 

The density of a block of wood is 100 kg/m^3. Can you use calculus to determine the total mass of the object shown?
Can you relate $\iiint \rho\, dx\, dy\, dz$ to $\int dm$?

Math II: Math vs Physics Reasoning

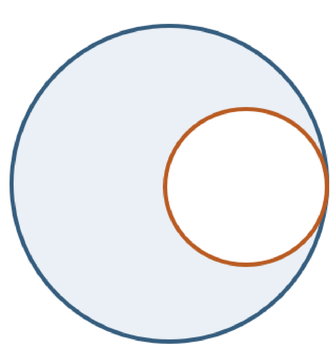
$$\int_1^2 \frac{2}{x^3} - x^2\, dx$$

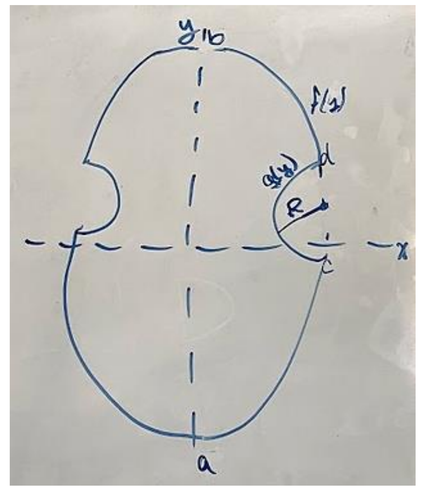
Comfortability with a lack of parentheses indicates thinking of the integral as a mathematical operator.

$$\int_1^2 \left(\frac{2}{x^3} - x^2\right) dx$$

The use of parentheses indicates thinking of the integral as adding up pieces, which better allows the student to apply physical meaning.

Math VI: Use of Geometrical Resources

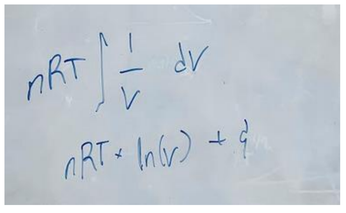




Example illustrating a student's unproductive application of geometrical resources.

Physics I vs Physics II: Mathematical Reasoning in a Physical Context

$$\int \frac{nRT}{V} dV$$



Students applied mathematical reasoning to the physical context without complication.

$$\int dm$$

Students used mathematical reasoning but were unable to apply appropriate physical meaning.

In this integral, m is mass. What do you think of this expression?

$$\int dm$$

Acknowledgements:
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References:
[1]Jones, "Understanding the integral: Students' symbolic forms", 2013
[2]Hammer et al., "Resources, framing, and transfer", 2004

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