Spring 2014 Ph.D. Qualifying Practice Exam, Signals and Systems

No reference materials, no calculator. Show all work for full credit!

1. Input voltage $x(t)$ applied to an inverting op-amp follower circuit produces output $y(t)$ according to

$$y(t+t_{\mathrm{p}}) = \begin{cases} -V_{\mathrm{ref}} & x(t) > V_{\mathrm{ref}} \\ V_{\mathrm{ref}} & x(t) < -V_{\mathrm{ref}} \\ -x(t) & \text{otherwise} \end{cases},$$

where op-amp reference voltage V_{ref} and propagation delay t_{p} are both positive constants. Answer the following questions YES or NO and provide justification for each answer.

(a) (2 pts) Is this system BIBO stable?

(b) (2 pts) Is the system causal?

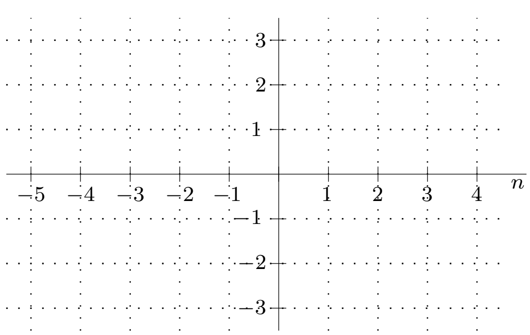
(c) (2 pts) Is the system invertible?

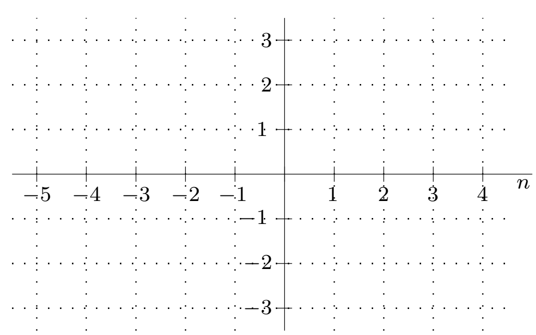
(d) (2 pts) Is the system linear?

(e) (2 pts) Is the system memoryless?

(f) (2 pts) Is the system time invariant?

2. (8 pts) An $(N = 5)$-periodic signal $x[n]$ is shown in Figure 1. Using the graphs provided, sketch $y[n] = x[1+3n]$ and $z[n] = x[2-n/2]$ over $-5 \leq n \leq 4$.





3. A real LTIC system with input $x(t)$ and output $y(t)$ is described by the following constant coefficient linear differential equation:

$$(D^3 + 9D)\{y(t)\} = (2D^3 + 1)\{x(t)\}.$$

(a) (2 pts) What is the characteristic equation of this system?

(b) (2 pts) What are the characteristic modes of this system?

(c) (8 pts) Assuming $y_{\text{zir}}(0) = 4$, $\dot{y}_{\text{zir}}(0) = -18$, and $\ddot{y}_{\text{zir}}(0) = 0$, determine this system's zero input response $y_{\text{zir}}(t)$. Simplify $y_{\text{zir}}(t)$ to include only real terms (i.e., no j's should appear in your answer).

4. (8 pts) Consider signals $x(t) = \sin(t)\,[u(t + 2\pi) - u(t + \pi)]$ and $h(t) = -u(t+2)+3u(t-1)-2u(t-\frac{5}{2})$. Determine the approximate time t_{min} where $y(t) = x(t) * h(t)$ is a minimum. Note, the minimum value of $y(t) \neq 0$!

5. A signal $x[n] = (0.5)^n \left(u[n+4] - u[n-4]\right)$ is input into a LTID system with an impulse response given by

$$h[n] = \begin{cases} 2 & \text{if } [(n \bmod 6) < 4] \text{ and } [n \geq 0] \\ 0 & \text{otherwise} \end{cases}$$

Recall, $(n \bmod p)$ is the remainder of the division n/p. The system is described according to the difference equation $y[n] - y[n-6] = 2x[n] + 2x[n-1] + 2x[n-2] + 2x[n-3]$.

(a) (5 pts) Determine the system's external stability. Mathematically justify your answer.

(b) (15 pts) Determine the value of $y[10]$, the zero-state output of system $h[n]$ in response to $x[n]$ at time $n = 10$. I'm looking for a single number, expressed in decimal form to at least three decimal places (e.g., $y[10] = 3.142$).

6. Consider the signals $x(t)$ and $y(t)$, as shown in Fig. 2.

   (a) (4 pts) Using the definition, compute $X(s)$, the bilateral Laplace transform of $x(t)$.

   (b) (7 pts) Using Laplace transform properties, express $Y(s)$, the bilateral Laplace transform of $y(t)$, as a function of $X(s)$, the bilateral Laplace transform of $x(t)$. Simplify as much as possible WITHOUT substituting your answer from part 6a.

7. Consider an LTIC system with system function $H(s) = \frac{-3s}{(s+0.5+3j)(s+0.5-3j)}$.

   (a) (3 pts) Compute the system output $y(t)$ in response to the input $x(t) = 1 + \sin(3t) + \cos(10t)$.

   (b) (6 pts) Sketch $|H(j\omega)|$ over $-10 \leq \omega \leq 10$.

8. Consider the LTID system shown in Fig. 3, where parameters c_1 and c_2 are constants.

   (a) (8 pts) Determine the system function $H(z)$, expressed in standard rational form.

   (b) (2 pts) What is the order N of this system?

   (c) (2 pts) Identify the locations of the N poles and N zeros of this system.

   (d) (6 pts) Determine c_1 and c_2 so that this system functions as an HPF with wide passband.

   (e) (2 pts) Is the realization shown in Fig. 3 canonical? Fully explain your answer.

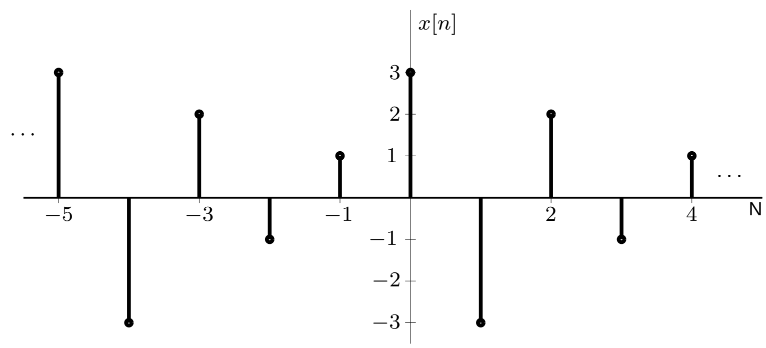


Figure 1: 5-periodic signal $x[n]$ for Prob. 2.

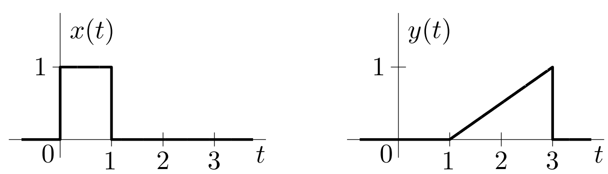


Figure 2: Signals $x(t)$ and $y(t)$ for Prob. 6.

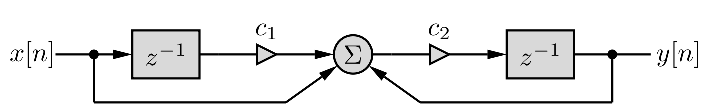


Figure 3: LTID system for Prob. 8.