Research Experience for Teachers: Mitigating Natural Disasters

NDSU | NORTH DAKOTA STATE UNIVERSITY

Lesson #3 The Mean and the Standard Deviation

Subject Area	Mathematics
Grade Level	9-12
Prior knowledge	Measures of center and spread for univariate data
Time required	2 class periods

Summary

The mean or average is a commonly reported statistic. However, without a measure of variability, the mean tells an incomplete or possibly misleading story about the data. The standard deviation is a statistic that tells you if the data are close to the mean or spread out over a large range. In this way, the standard deviation allows you to effectively compare two or more data sets. In this lesson students will calculate standard deviation both by hand and using a graphing calculator.

Education Standard

NCTM Principals and Standards

Select and use appropriate statistical methods to analyze data

- Find, use, and interpret measures of center and spread, including mean and interquartile range.

I can statement

I can describe data using the Mean and Standard Deviation.

Introduction

Dr. Nic's Math and Stats video: *Understanding Summary Statistics: Measures of Spread*
https://creativemaths.net/videos/video-spread/

Notes and Examples

Variance and **Standard Deviation** are measures of spread that are based on the **Mean**.

Population Variance:

$$\sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \mu)^2}{n}$$

Sample Variance:

$$s^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}$$

The **Standard Deviation (s)** is the square root of the **Variance (s^2)**.

Steps to calculate the sample standard deviation:

1. Calculate the Mean. $\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$

2. Calculate the Deviation of each observation from the Mean. $\left(x_i - \bar{x}\right)$

3. Square each Deviation. $\left(x_i - \bar{x}\right)^2$

4. Sum the Squared Deviations. $\sum_{i=1}^{n} \left(x_i - \bar{x}\right)^2$

5. Divide the sum of the squared deviations by (n-1). $s^2 = \frac{\sum_{i=1}^{n} \left(x_i - \bar{x}\right)^2}{n-1}$

6. Take the square root. $s = \sqrt{s^2}$

The standard deviation measures, on average, how much the observations deviate from the mean. A small standard deviation indicates that the observations are clustered around the mean. A large standard deviation indicates that the observations are spread out or in other words, a large range.

Calculating by hand

Calculate the standard deviation for the following observations: 1, 2, 3, 3, 4, 5, 5, 9

1. Calculate the mean. $\bar{x}$ = _____

2. Complete the table.

3. Calculate the variance. s^2 = _____

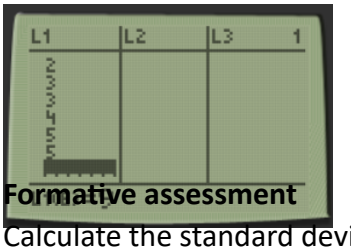
4. Take the square root of variance. s = _____

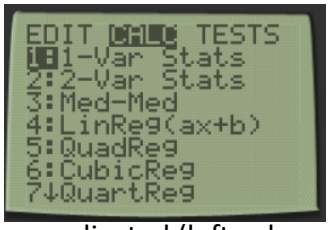
Observation (x)	$x - \bar{x}$	$\left(x - \bar{x}\right)^2$
Sum		

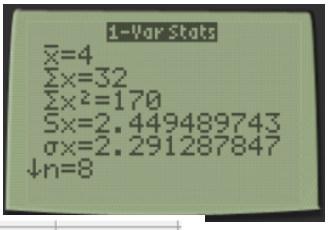
Calculating with the TI graphing calculator

STAT → EDIT Enter data in a list.

STAT→ CALC→ 1-Var Stats ENTER







Formative assessment

Calculate the standard deviation of the unadjusted (left column) data.

s = _____

Without calculating, what would you expect for the standard deviation of the adjusted (right column) data. Make a prediction.

Median	Median Fixed
69	112
56	93
54	89
52	82
47	47
46	46
46	46
45	45
43	43
36	36
35	35
34	34
31	31

Answers to formative assessment

Calculate the standard deviation of the unadjusted (left column) data.

s = 10.52

Without calculating, what would you expect for the standard deviation of the adjusted (right column) data. Make a prediction.

Since the range or variability of the right column is greater, the standard deviation will be greater than 10.52

Median	Median Fixed
69	112
56	93
54	89
52	82
47	47
46	46
46	46
45	45
43	43
36	36
35	35
34	34
31	31

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Acknowledgements

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Exercise #3 The Mean and Standard Deviation

The Data

The data in for this exercise was collected from a model of the intersection of Labeaux and County Road 18 in Albertville MN created using PTV VISSIM software. Videos of traffic simulation which generated the data:

0% AV saturation: https://www.youtube.com/watch?v=IkwWuVd-EAI
90% AV saturation: https://www.youtube.com/watch?v=S3Czr_taME0
3D view: https://www.youtube.com/watch?v=ylzQreGefqY

The Variables

The software collected data on the following variables:

Average Vehicle Delay is the average number of seconds a vehicle is stopped at the intersection.

Average Queue Length is the average length in meters of the line of vehicles stopped.

Maximum Queue Length is the maximum length in meters of the line of vehicles stopped.

Number of Queue Stops is the number of stops made by all vehicles at the intersection.

These variables are measured at each entrance to the intersection:

Westbound **(WB)**

Southbound **(SB)**

Eastbound **(EB)**

Northbound **(NB).**

Each simulation lasted 90 minutes (5400 seconds) with data collected every 15 minutes (900 seconds) and the first and last 15-minute intervals were discarded. Therefore, the data is collected for four time intervals for each run of the simulation:

900-1800 seconds, **1800-2700** seconds, **2700-3600** seconds, **3600-4500** seconds

Each variable is measured for seven autonomous vehicle saturation rates:

0% AV – all human driven cars

15% AV – 85% human driven cars

30% AV – 70% human driven cars

45% AV – 55% human driven cars

60% AV – 40% human driven cars

75% AV – 25% human driven cars

90% AV – 10% human driven cars

At each AV saturation rate, the simulation was run ten times. As a result, at each variable at each AV saturation rate there are 160 observations. One for each of the four directions at each of the four time intervals for each of the ten simulations.

At each saturation rate, the autonomous vehicles are tested using three different driving behaviors:

Cautious, **Normal**, **Aggressive**

	Queue stops	**SB**	**0% AV**	**90%AV**	**90%AV**	**90%AV**
				Cautious	**Normal**	**Aggressive**
Run	TIME INT		QSTOPS	QSTOPS	QSTOPS	QSTOPS
1	900-1800	2	312	436	297	235
1	1800-2700	2	299	390	284	269
1	2700-3600	2	234	316	223	220
1	3600-4500	2	247	290	243	231
2	900-1800	2	258	302	246	239
2	1800-2700	2	258	322	246	232
2	2700-3600	2	285	385	274	260
2	3600-4500	2	303	439	279	258
3	900-1800	2	271	316	259	249
3	1800-2700	2	291	446	270	261
3	2700-3600	2	249	290	242	235
3	3600-4500	2	233	281	222	214
4	900-1800	2	228	307	213	211
4	1800-2700	2	273	318	246	229
4	2700-3600	2	289	707	282	274
4	3600-4500	2	329	1594	296	241
5	900-1800	2	267	330	256	240
5	1800-2700	2	248	332	237	222
5	2700-3600	2	280	441	267	246
5	3600-4500	2	312	587	294	280
6	900-1800	2	254	276	240	227
6	1800-2700	2	243	338	253	224
6	2700-3600	2	301	342	282	258
6	3600-4500	2	246	323	238	234
7	900-1800	2	311	479	276	247
7	1800-2700	2	262	448	259	248
7	2700-3600	2	289	320	266	246
7	3600-4500	2	275	335	257	247
8	900-1800	2	238	289	228	221
8	1800-2700	2	252	339	245	234
8	2700-3600	2	306	426	284	265
8	3600-4500	2	285	742	274	266
9	900-1800	2	240	244	238	228
9	1800-2700	2	264	361	255	244
9	2700-3600	2	269	364	257	245
9	3600-4500	2	233	313	228	220
10	900-1800	2	244	411	239	231

10	1800-2700	2	278	351	256	241
10	2700-3600	2	265	502	250	235
10	3600-4500	2	277	573	261	253

Using the southbound queue stop data, construct a histogram for the human driven vehicles (0% AV) and a histogram for each of the three driving behaviors of autonomous vehicles at 90% saturation rate. Report the median, mean and standard deviation.

0% autonomous vehicles

Median: _____

Mean: _____

Standard Deviation: _____

90% AV – Cautious

Median: _____

Mean: _____

Standard Deviation: _____

90% AV – Normal

Median: _____

Mean: _____

Standard Deviation: _____

90% AV – Aggressive

Median: _____

Mean: _____

Standard Deviation: _____

Exercise#3 The Mean and Standard Deviation Answers

Using the southbound queue stop data, construct a histogram for the human driven vehicles (0% AV) and a histogram for each of the three driving behaviors of autonomous vehicles at 90% saturation rate. Report the median, mean and standard deviation.

0% autonomous vehicles

Median: 268

Mean: 269.95

Standard Deviation: 26.26

90% AV – Cautious

Median: 340.5

Mean: 415.125

Standard Deviation: 221

90% AV – Normal

Median: 256

Mean: 256.55

Standard Deviation: 21.32

90% AV - Aggressive

Median: 240.5

Mean: 241.5

Standard Deviation: 16.71