Research Experience for Teachers: Mitigating Natural Disasters

NDSU | NORTH DAKOTA STATE UNIVERSITY

Lesson #5 The Standard Normal Distribution

Subject Area	Mathematics
Grade Level	9-12
Prior knowledge	Measures of center and spread for univariate data, Shape of a distribution.
Time required	2 class period

Summary

The Normal distribution has a specific symmetric bell shape that follows the 68-95-99.7% rule. The Standard Normal distribution has an additional property: the mean is set to 0 and the standard deviation is set to 1. This is important because standardizing a metric allows for comparisons with other metrics that have also been standardized. This process allows us to fairly compare raw data measured on different scales. In this lesson students will calculate z-scores and calculate probability using the standard normal distribution.

Education Standard

NCTM Principals and Standards

Select and use appropriate statistical methods to analyze data

- Recognize how linear transformations of univariate data affect shape, center, and spread.

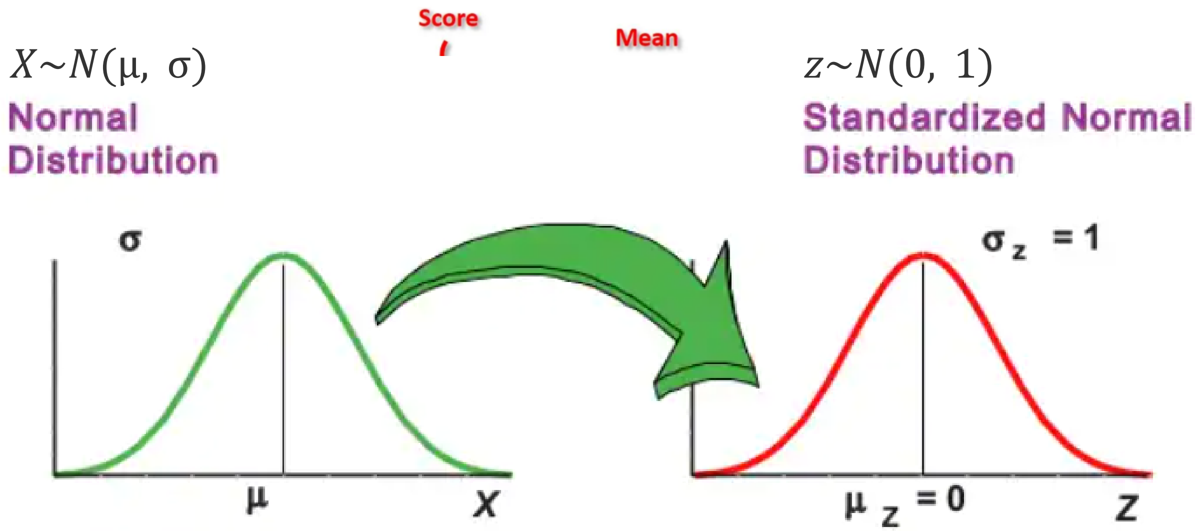
I can statement

I can calculate a z-score and calculate probability using a Standard Normal Distribution table.

Notes and Examples

The Standard Normal Distribution is a Normal distribution with mean = 0 and standard deviation = 1.

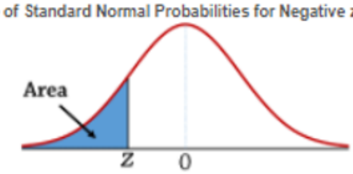
To standardize, calculate a z-score for the observation.



Because we can transform any normal random variable into a standard normal random variable, we only need one table.

1.

Table of Standard Normal Probabilities for Negative z-scores



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423

The table entry for z is the area under the curve to the left of z.

$P(z < -3.01) =$ ______________

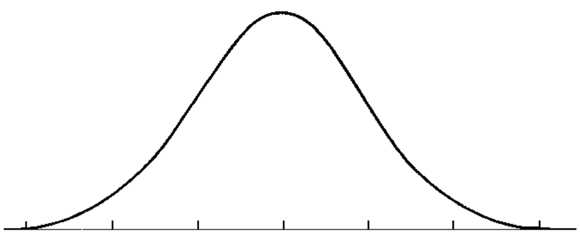
$P(z < -2.55) =$ ______________

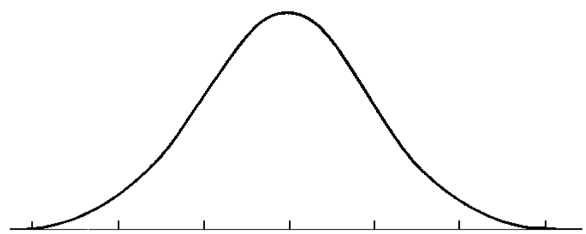
$P(z < -1.76) =$ ______________

$P(z < -1.03)$ = ____________

2. Calculate the probability that z is between -0.5 and 1.5

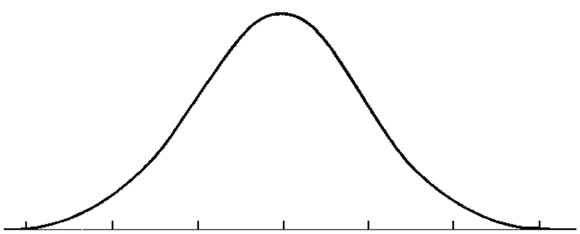
$P(-0.5 < z < 1.5) = P(z < 1.50) - P(z < -0.5)$





$P(z < 1.50)$ = __________________

$P(z < -0.5)$ = ______________________



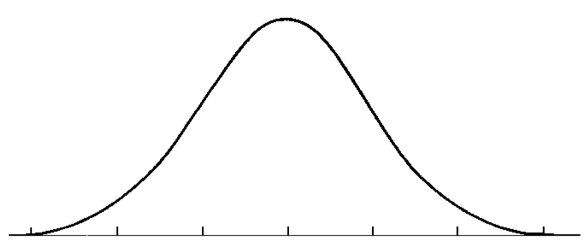
$P(-0.5 < z < 1.5)$ =

____

3. A study shows that the number of hours of sleep per night for a U.S. adult between the ages of 18 and 65 is normally distributes with a mean of 6.8 hours and a standard deviation of 0.6 hours.

Let the random variable X be the number of hours of sleep per night for a randomly selected U.S. adult between the ages of 18 and 65. Then $X \sim N(6.8,\ 0.6)$

a) Label the axis of the normal distribution using the mean and standard deviation from the example. Then shade the region that represents the portion of U.S. adults who get less than 6 hours of sleep per night. This region is described by the expression $P(X < 6)$.



b) To calculate the probability $P(X < 6)$, standardize the observed value $x_i = 6$ using $z = \frac{x_i - \mu}{\sigma}$

c) $P\left(z < \frac{6-6.8}{0.6}\right) = P(z < \ -1.33) =$ ____________________

Formative assessment

Suppose that the birth weight of newborn babies in the U.S. is normally distributed with a mean of 7 pounds and a standard deviation of 1.6 pounds.

Let the random variable X be the weight of a randomly selected record of live birth. Then $X \sim N(7,\ 1.6)$

a) What is the probability that a randomly selected record shows a birth weight of less than 5 pounds?

b) What is the probability that a randomly selected record shows a birth weight between 6 and 9 pounds?

Answers to formative assessment

Suppose that the birth weight of newborn babies in the U.S. is normally distributed with a mean of 7 pounds and a standard deviation of 1.6 pounds.

Let the random variable X be the weight of a randomly selected record of live birth. Then $X \sim N(7,\ 1.6)$

a) What is the probability that a randomly selected record shows a birth weight of less than 5 pounds?

$P(X < 5) = P\left(z < \frac{5-7}{1.6}\right) = P(z < -1.54) = 0.0618 = 6.18\%$

b) What is the probability that a randomly selected record shows a birth weight between 6 and 9 pounds?

$P(6 < x < 9) = P(x < 9) - P(x < 6)$ = 20.42%

= $P\left(z < \frac{9-7}{1.6}\right) - P(z < \frac{6-7}{1.6})$

= $P(z < 1.54) - P(z < -0.625)$

= 0.9382 – 0.734
= 0.2042

Author

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Exercise #5 The Standard Normal Distribution

<u>The Data</u>:

The data in for this exercise was collected from a model of the intersection of Labeaux and County Road 18 in Albertville MN created using PTV VISSIM software. Videos of traffic simulation which generated the data:

0% AV saturation: https://www.youtube.com/watch?v=IkwWuVd-EAI

90% AV saturation: https://www.youtube.com/watch?v=S3Czr_taME0

3D view: https://www.youtube.com/watch?v=ylzQreGefqY

<u>The Variables</u>:

The software collected data on the following variables:

Average Vehicle Delay is the average number of seconds a vehicle is stopped at the intersection.

Average Queue Length is the average length in meters of the line of vehicles stopped.

Maximum Queue Length is the maximum length in meters of the line of vehicles stopped.

Number of Queue Stops is the number of stops made by all vehicles at the intersection.

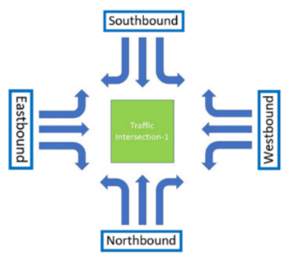
These variables are measured at each entrance to the intersection:

Westbound **(WB)**

Southbound **(SB)**

Eastbound **(EB)**

Northbound **(NB).**



Each simulation lasted 90 minutes (5400 seconds) with data collected every 15 minutes (900 seconds) and the first and last 15-minute intervals were discarded. Therefore, the data is collected for four time intervals for each run of the simulation:

900-1800 seconds, **1800-2700** seconds, **2700-3600** seconds, **3600-4500** seconds

Each variable is measured for seven autonomous vehicle saturation rates:

0% AV – all human driven cars

15% AV – 85% human driven cars

30% AV – 70% human driven cars

45% AV – 55% human driven cars

60% AV – 40% human driven cars

75% AV – 25% human driven cars

90% AV – 10% human driven cars

At each AV saturation rate, the simulation was run ten times. As a result, at each variable at each AV saturation rate there are 160 observations. One for each of the four directions at each of the four time intervals for each of the ten simulations.

At each saturation rate, the autonomous vehicles are tested using three different driving behaviors:

Cautious, **Normal**, **Aggressive**

	Queue Stops								
	0% AV								
Run	TIMEINT	WB	QSTOPS	SB	QSTOPS	EB	QSTOPS	NB	QSTOPS
1	900-1800	1	123	2	312	3	113	4	274
1	1800-2700	1	104	2	299	3	100	4	279
1	2700-3600	1	127	2	234	3	124	4	297
1	3600-4500	1	157	2	247	3	142	4	228
2	900-1800	1	139	2	258	3	173	4	274
2	1800-2700	1	184	2	258	3	116	4	269
2	2700-3600	1	152	2	285	3	119	4	263
2	3600-4500	1	141	2	303	3	102	4	208
3	900-1800	1	117	2	271	3	134	4	252
3	1800-2700	1	116	2	291	3	117	4	235
3	2700-3600	1	107	2	249	3	134	4	280
3	3600-4500	1	126	2	233	3	110	4	207
4	900-1800	1	105	2	228	3	103	4	274
4	1800-2700	1	135	2	273	3	120	4	271
4	2700-3600	1	110	2	289	3	141	4	297
4	3600-4500	1	131	2	329	3	121	4	257
5	900-1800	1	176	2	267	3	122	4	301
5	1800-2700	1	138	2	248	3	131	4	262
5	2700-3600	1	167	2	280	3	128	4	278
5	3600-4500	1	141	2	312	3	139	4	289
6	900-1800	1	128	2	254	3	117	4	237
6	1800-2700	1	104	2	243	3	102	4	294
6	2700-3600	1	145	2	301	3	95	4	249
6	3600-4500	1	157	2	246	3	124	4	294
7	900-1800	1	107	2	311	3	139	4	280
7	1800-2700	1	148	2	262	3	135	4	320
7	2700-3600	1	113	2	289	3	102	4	253
7	3600-4500	1	132	2	275	3	108	4	267
8	900-1800	1	148	2	238	3	120	4	285
8	1800-2700	1	130	2	252	3	165	4	294
8	2700-3600	1	143	2	306	3	129	4	250
8	3600-4500	1	122	2	285	3	158	4	255
9	900-1800	1	141	2	240	3	150	4	246
9	1800-2700	1	133	2	264	3	144	4	275
9	2700-3600	1	140	2	269	3	145	4	328
9	3600-4500	1	143	2	233	3	120	4	256
10	900-1800	1	125	2	244	3	165	4	257
10	1800-2700	1	135	2	278	3	144	4	258
10	2700-3600	1	132	2	265	3	122	4	289

10	3600-4500	1	134	2	277	3	127	4	239

Using the number of queue stops data, for the westbound and southbound traffic:

1. Enter data in List 1, List 2, List 3, and List 4.
   - L1: WB queue stops
   - L2: z-score for WB data
   - L3: SB queue stops
   - L4: z-score for SB data

2. Construct a histogram for each L2 and L4.

3.

4. What is the probability west bound traffic has between 130 – 150 stops during a 15-minute interval?

5. What is the probability south bound traffic has more than 300 stops during a 15-minute interval?

	Queue Stops		0% AV				
			List 1	**List 2**		**List 3**	**List 4**
Run	TIME INT	WB	QSTOPS	z_{WB}	SB	QSTOPS	z_{SB}
1	900-1800	1	123		2	312	
1	1800-2700	1	104		2	299	
1	2700-3600	1	127		2	234	
1	3600-4500	1	157		2	247	
2	900-1800	1	139		2	258	
2	1800-2700	1	184		2	258	
2	2700-3600	1	152		2	285	
2	3600-4500	1	141		2	303	
3	900-1800	1	117		2	271	
3	1800-2700	1	116		2	291	
3	2700-3600	1	107		2	249	
3	3600-4500	1	126		2	233	
4	900-1800	1	105		2	228	
4	1800-2700	1	135		2	273	
4	2700-3600	1	110		2	289	
4	3600-4500	1	131		2	329	
5	900-1800	1	176		2	267	
5	1800-2700	1	138		2	248	
5	2700-3600	1	167		2	280	
5	3600-4500	1	141		2	312	
6	900-1800	1	128		2	254	
6	1800-2700	1	104		2	243	
6	2700-3600	1	145		2	301	
6	3600-4500	1	157		2	246	
7	900-1800	1	107		2	311	
7	1800-2700	1	148		2	262	
7	2700-3600	1	113		2	289	
7	3600-4500	1	132		2	275	
8	900-1800	1	148		2	238	
8	1800-2700	1	130		2	252	
8	2700-3600	1	143		2	306	
8	3600-4500	1	122		2	285	
9	900-1800	1	141		2	240	
9	1800-2700	1	133		2	264	
9	2700-3600	1	140		2	269	
9	3600-4500	1	143		2	233	
10	900-1800	1	125		2	244	
10	1800-2700	1	135		2	278	

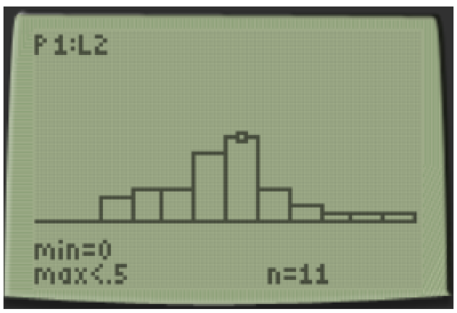
10	2700-3600	1	132		2	265	
10	3600-4500	1	134		2	277	

Exercise#5 The Standard Normal Distribution Answers

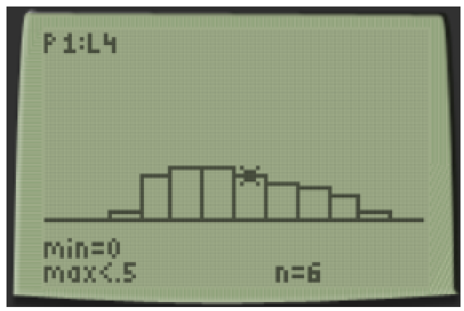
Using the number of queue stops data at 0% AV, for westbound and southbound traffic:

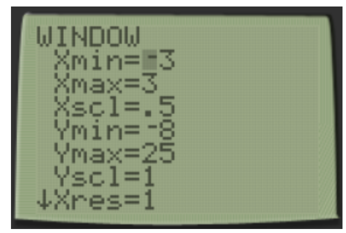
1. Enter data in List 1, List 2, List 3, and List 4.
   - L1: WB queue stops
   - L2: z-score for WB data
   - L3: SB queue stops
   - L4: z-score for SB data
2. Construct a histogram for each L2 and L4.

Westbound:



Southbound:





3. What is the probability westbound traffic has between 130 – 150 stops during a 15-minute interval?

$P(130 < X < 150) = P(X < 150) - P(X < 130)$

$P\left(z < \frac{150-133.9}{18.875}\right) - P\left(\frac{130-133.9}{18.875}\right)$

$= P(z < 0.85) - P(z < -0.21)$

= 0.8023 – 0.4168

= 0.3855 = 38.55%

4. What is the probability southbound traffic has more than 300 stops during a 15-minute interval?

$P(X > 300) = 1 - P(X < 300) = 1 - P\left(z < \frac{300-269.95}{26.26}\right)$

$= 1 - P(z < 1.14)$

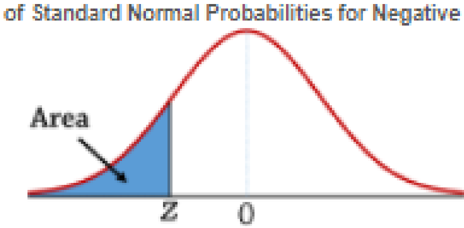
= 1 - 0.8729

= 0.1271 = 12.71%

	Queue Stops		**0% AV**				
Run	TIME INT	WB	QSTOPS	z_{WB}	SB	QSTOPS	z_{SB}
1	900-1800	1	123	-0.57748	2	312	1.601295
1	1800-2700	1	104	-1.58411	2	299	1.106245
1	2700-3600	1	127	-0.36556	2	234	-1.369
1	3600-4500	1	157	1.223841	2	247	-0.87395
2	900-1800	1	139	0.270199	2	258	-0.45506
2	1800-2700	1	184	2.654305	2	258	-0.45506
2	2700-3600	1	152	0.95894	2	285	0.573115
2	3600-4500	1	141	0.376159	2	303	1.258568
3	900-1800	1	117	-0.89536	2	271	0.039985
3	1800-2700	1	116	-0.94834	2	291	0.801599
3	2700-3600	1	107	-1.42517	2	249	-0.79779
3	3600-4500	1	126	-0.41854	2	233	-1.40708
4	900-1800	1	105	-1.53113	2	228	-1.59749
4	1800-2700	1	135	0.058278	2	273	0.116146
4	2700-3600	1	110	-1.26623	2	289	0.725438
4	3600-4500	1	131	-0.15364	2	329	2.248667
5	900-1800	1	176	2.230464	2	267	-0.11234
5	1800-2700	1	138	0.217219	2	248	-0.83587
5	2700-3600	1	167	1.753642	2	280	0.382711
5	3600-4500	1	141	0.376159	2	312	1.601295
6	900-1800	1	128	-0.31258	2	254	-0.60739
6	1800-2700	1	104	-1.58411	2	243	-1.02628
6	2700-3600	1	145	0.588079	2	301	1.182407

6	3600-4500	1	157	1.223841	2	246	-0.91203
7	900-1800	1	107	-1.42517	2	311	1.563214
7	1800-2700	1	148	0.74702	2	262	-0.30274
7	2700-3600	1	113	-1.10728	2	289	0.725438
7	3600-4500	1	132	-0.10066	2	275	0.192308
8	900-1800	1	148	0.74702	2	238	-1.21668
8	1800-2700	1	130	-0.20662	2	252	-0.68355
8	2700-3600	1	143	0.482119	2	306	1.37281
8	3600-4500	1	122	-0.63046	2	285	0.573115
9	900-1800	1	141	0.376159	2	240	-1.14052
9	1800-2700	1	133	-0.04768	2	264	-0.22658
9	2700-3600	1	140	0.323179	2	269	-0.03618
9	3600-4500	1	143	0.482119	2	233	-1.40708
10	900-1800	1	125	-0.47152	2	244	-0.98819
10	1800-2700	1	135	0.058278	2	278	0.30655
10	2700-3600	1	132	-0.10066	2	265	-0.1885
10	3600-4500	1	134	0.005298	2	277	0.268469
			$\bar{x}$ = 133.9	s=18.875		$\bar{x}$=269.95	s = 26.26

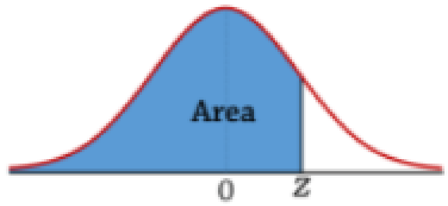
Table of Standard Normal Probabilities for Negative z-scores



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
-0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

Figure 1. Table of Standard Normal Probabilities for Negative z-scores. Table entries are "less than" areas.

Table of Standard Normal Probabilities for Positive z-scores



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

Figure 2. Table of Standard Normal Probabilities for Positive z-scores. Table entries are "less than" areas.