Slope Failure and Calculus

Content Area(s)/Course/Grade: AP Calculus	Unit: Techniques of Integration
Lesson Topic: Properties of Definite Integrals and continued FTC	Length of Lesson: 2 Days
Materials for Students: Calculator and writing utensil	Materials for Teacher: Grid plates and foam cutouts, guided worksheet/notes/presentation

Standard(s) Addressed:
Applying Properties of Definite Integrals
Finding the Average Value of a Function on an Interval
The Fundamental Theorem of Calculus and Definite Integrals

Student Outcome(s):
I can apply properties of integrals to simplify and help solve them
I can find the average value of a function
I can find when a function is equal to its average

Context for Learning

This lesson will focus on some useful properties of definite integrals. It will also show a powerful method for finding average value of a continuous function. This has some nice connections to slope failures and rainfall that they will get to explore in their activity.

Instructional Delivery

Lesson notes: Day 1 of this lesson will be mostly teacher lead discussion and notes about the 3 student outcomes. Notes will begin with a quick discussion about rainfall and how we often generalize it with averages. This should lead to the idea of finding the average of any function. We will first explore some properties that we will need to find first. There are 3 main properties that will be addressed and practiced. We will then be able to find the average value of a function and when the function is equal to that average value which is called the Mean Value Theorem for Integrals. Day 2 of the lesson will be a hands-on activity having the students apply these concepts to more examples of slope failure and rainfall data. After activity is completed students will be able to continue practicing the concept on their worksheet. (notes are attached)

Activity: Students will work in small groups and evaluate definite integrals based on some of their properties. They will also evaluate the average value of slope failure land masses thus getting average elevation and average value of rainfall. They will also find when the function is equal to the average value. They will have to use a variety of techniques to find values in the questions. They will have to use Geometric formulas, definite integral properties, FTC from a given function, and FTC from a set of points they turn into a function. (activity worksheet is attached)

Assessment/Evaluation (Formative/Summative)

There will be an informal formative assessment in the form of their worksheet. Gather how they are doing by walking around to each student and observing them work. There will be a formal formative assessment the day after this lesson in the form of a mini quiz that will be one property question and one average value question. (worksheet and mini quiz will be attached with this lesson plan)

Accommodations: Make sure everyone has a calculator for this worksheet and activity. Reduce the amount of question for those who need it. Walk around and help those students that need more help. Extra time for students that need it on the mini quiz.

Slope Failures and Calculus
Lesson 4 Properties and Average Value

Objectives: To be able to apply properties of definite integrals. To be able to find the average value of a function and when the function is equal to that average.

Properties of definite integrals:

$\int_a^a f(x)dx = 0$ $\int_b^a f(x)dx = -\int_a^b f(x)dx$

$\int_a^c f(x)dx = \int_a^b f(x)dx + \int_b^c f(x)dx$, given b is between a and c (is this part needed?)

Given: $\int_0^5 f(x)dx = 10 \; and \; \int_5^7 f(x)dx = 3$

Evaluate:

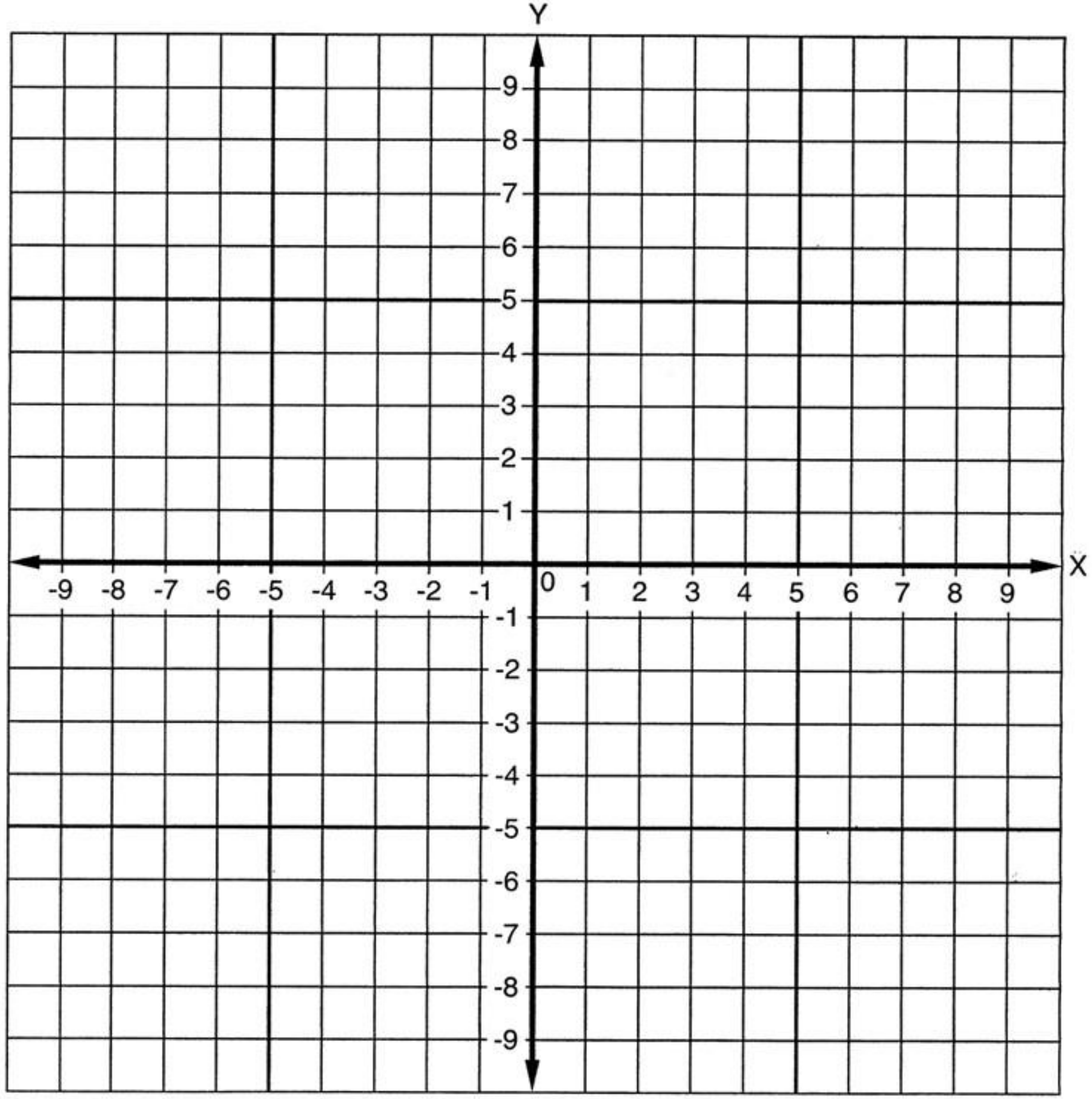
$\int_0^7 f(x)dx$

$\int_0^0 f(x)dx$

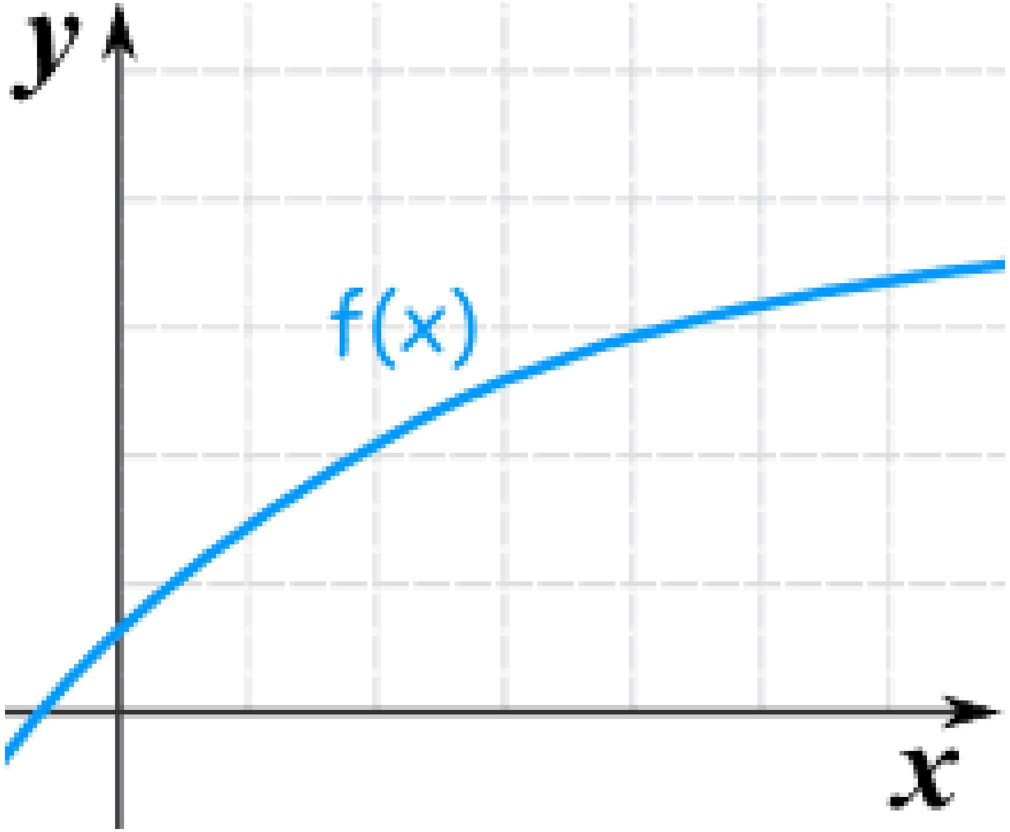
$\int_7^0 f(x)dx$

$\int_5^7 6f(x)dx$

$$\int_{-5}^{5} \sqrt{25-x^2}\,dx + \int_{-5}^{5} -\sqrt{25-x^2}\,dx$$



To better understand the mean value theorem for integrals we will look at a couple diagrams.



Average Value of a function on an interval.

f must be integrable on a closed interval [a,b], then the average value of f on [a,b] is:

Average Value =

The Mean Value Theorem for Integrals

If f is continuous on a closed interval [a,b], then there exists at least one number c in the closed interval [a,b] such that:

$$f(c) = \frac{1}{b-a}\int_a^b f(x)dx$$

Find the average value of $f(x) = 3x^2 - 2x \; on \; [1,4]$ and find a c such that f(c)=average value.

Find the average value of f(x) and apply the MVT for integrals for f(x) given

$$f(x) = \cos x \; on \; \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

Slope Failures and Calculus Lesson 4 Fundamental Theorem of Calculus

Hand on Activity Lab 1 Names in the group:_______________________

Slope failures happen across the world and often have terrible effects. We will learn more about slope failures as this unit progresses and how there is lots of calculus connected to the concept. In today activity your group you will need to work together, discuss possible techniques, and find the area of the foam sheets using the plastic sheet and the coordinate plane on it. The foam sheets now represent land area or rainfall data. Today we will use the FTC and the MVT to find the exact area of the foam sheet, the average value, when you equal the average value, and explain the answer within the context. You will need to use a graphing calculator today for one of the questions to make a quadratic equation.

Foam sheet 1 (color)

Use the FTC to find the area of the foam sheet and explain what the area represents. Use the MVT to find the average value of the foam sheet and when it equals the average value, and lastly interpret the results. (note any markings on the foam)

Foam sheet 2 (color)

Use the FTC to find the area of the foam sheet and explain what the area represents. Use the MVT to find the average value of the foam sheet and when it equals the average value, and lastly interpret the results. (note any markings on the foam and you will need to use your calculator to make a cubic function ask for help if you get stuck)

Can you think of a time in your daily life where you might see the MVT?

SF and Calc Lesson 4 WS   Name:____________________________

1. Evaluate the integral using the following values: $\int_2^4 x^3 dx = 60$, $\int_2^4 x dx = 6$, $\int_2^4 dx = 2$.

a. $\int_4^2 x dx$

c. $\int_2^4 8x dx$

e. $\int_2^4 (6+2x-x^3) dx$

b. $\int_2^2 x^3 dx$

d. $\int_2^4 (x-8) dx$

2. Given $\int_0^5 f(x) dx = 10$ and $\int_5^7 f(x) dx = 3$ evaluate:

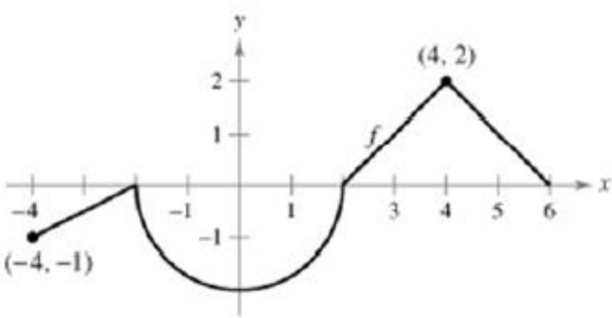
a. $\int_0^7 f(x) dx$

b. $\int_5^0 f(x) dx$

c. $\int_5^5 f(x) dx$

d. $\int_0^5 3f(x) dx$

3. Use the graph of f to determine the definite integrals



(a) $\int_0^2 f(x)\, dx$

(b) $\int_2^6 f(x)\, dx$

(c) $\int_{-4}^2 f(x)\, dx$

(d) $\int_{-4}^6 f(x)\, dx$

(e) $\int_{-4}^6 |f(x)|\, dx$

(f) $\int_{-4}^6 [f(x) + 2]\, dx$

4. Find the value(s) of c guaranteed by the MVT for integrals for the function over the given interval. Provide a sketch to support your answer.

a. $f(x) = x - 2\sqrt{x}$ on $[0,2]$

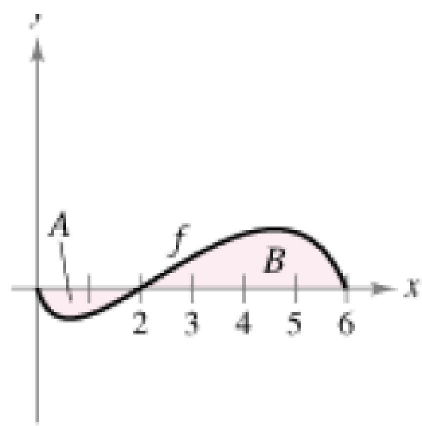
b. $g(x) = \cos x$ on $[-\pi/3, \pi/3]$

5. Find the average value of the function over the given interval and all values of x in the interval for which the function equals its average value. Provide a sketch to support your answer.

a. $f(x) = 4 - x^2$ on [-2, 2]

b. $f(x) = \sin x, [0, \pi]$

6. Use the graph of f and the fact that the area of region A is 1.5 and $\int_0^6 f(x)dx = 3.5$ to answer the following



a. $\int_0^2 f(x)dx$

b. $\int_2^6 f(x)dx$

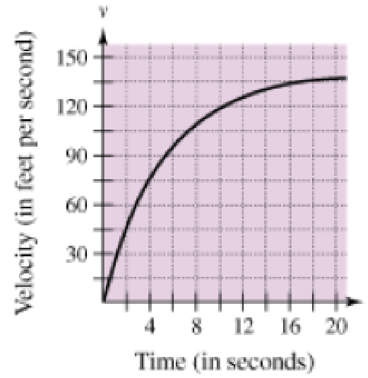
c. $\int_0^6 |f(x)|dx$

d. $\int_0^2 -2f(x)dx$

e. $\int_0^6 [2 + f(x)]dx$

f. Find the average value of f over [0, 6]

7. The graph shows the velocity of a car accelerating from rest. Use the graph to estimate the distance the car travels in 8 seconds.



8. Consider the function f that is continuous on the interval [-5,5] and for which

$\int_0^5 f(x)dx = 4$

Evaluate each integral:

a. $\int_0^5 [f(x)+2]dx$ b. $\int_{-2}^3 f(x+2)dx$ c. $\int_{-5}^5 f(x)dx$, f is even d. $\int_{-5}^5 f(x)dx$, f is odd.

SF and Calc Lesson 4 Mini Quiz   Name: ________________

Find the following definite integrals if we know

$\int_0^5 f(x)dx = 10$ and $\int_5^7 f(x)dx = 3.$

a.) $\int_0^7 f(x)dx$ b.) $\int_5^0 f(x)dx$ c.) $\int_5^5 f(x)dx$ d.) $\int_0^5 3f(x)dx$

Find the average value of the following function, $f(x) = 3x^2 - 2x + 4$, on the interval [2,4].

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