Slope Failure and Calculus

Content Area(s)/Course: AP Calculus	Unit: Techniques of Integration
Lesson Topic: Integration by Substitution and 2nd FTC	Length of Lesson: 2-3 Day
Materials for Students: Calculator and writing utensil	Materials for Teacher: guided worksheet/notes/presentation

Standard(s) Addressed:
Integrating using Substitution
The Fundamental Theorem of Calculus and Definite Integrals

Student Outcome(s):
I can evaluate a definite integral using Substitution
I can apply the 2nd FTC

Context for Learning

This is a natural ending to a unit on techniques of integration that will not have a strong connection to the slope failure and rain fall focus that most of the unit did. This lesson however is very important as most integration will need to use substitution to solve. The 2nd FTC is a lesser used concept but does help with strange problems and also builds some foundation toward learning about the next unit on derivative and integrals of exponential and logarithmic function.

Instructional Delivery

Lesson notes: The notes will cover how to perform u-substitution on indefinite and definite integrals and the 2nd FTC on basic integrals. Majority of the time will be spent with u-substitution. I expect the notes portion of the lesson to take about 1 class period and the independent work time on the worksheet to take about 1 class period. It is very important for students to practice these independently but have the resource of being able to ask you about them. (notes will be attached with this lesson plan)

Activity: No activity for this lesson

Assessment/Evaluation (Formative/Summative)

There will be an informal formative assessment in the form of their worksheet. Gather how they are doing by walking around to each student and observing them work. There will be a formal formative assessment the day after this lesson in the form of a mini quiz that will be one u-substitution questions and one 2nd FTC question. (worksheet and mini quiz will be attached with this lesson plan)

Accommodations: Make sure everyone has a calculator for this worksheet. Reduce the amount of question for those who need it. Walk around and help those students that need more help. Extra time for students that need it on the mini quiz.

Lesson 6 Integration by Substitution and 2^{nd} FTC

Objectives: To be able to use pattern recognition to find an indefinite integral. To be able to use a change of variables to find an indefinite or definite integral. To be able to evaluate a definite integral involving even or odd functions. To be able to apply the 2^{nd} FTC.

Integrating composite functions

Pattern Recognition – performing substitution mentally

Change of Variables – write the substitution steps

We are working with composite functions so that was the chain rule for differentiation which is:

$$\frac{d}{dx}f\big(g(x)\big) =$$

So it would be safe to say

$$\int f'\big(g(x)\big)g'(x)dx =$$

$\int (x^2 + 1)^2 (2x) dx$

$\int 5 \cos(5x)\, dx$

Sometimes we will have to multiply or divide by a constant to make it work.

$\int \sqrt{2x - 1}dx$

$$\int (x^2 + 4x)^3 (x + 2) dx$$

$$\int x\sqrt{3x+1}dx$$

$$\int \sin^2(3x)\cos(3x)\,dx$$

Definite Integrals by substitution

$$\int_0^1 x(x^2+1)^3 dx$$

$$\int_1^5 \frac{x}{\sqrt{2x-1}}\,dx$$

$$\int_0^{\frac{\pi}{2}} \cos\left(\frac{2x}{3}\right) dx$$

Second Fundamental Theorem of Calculus

If f is continuous on an open interval I containing a then for every x in the interval

$$\frac{d}{dx}\left[\int_a^x f(t)dt\right] = f(x)$$

$$\frac{d}{dx}\left[\int_4^x 5z^3 dz\right]$$

$$\frac{d}{dx}\left[\int_2^x \sqrt{t^2+1}\,dt\right]$$

$$\frac{d}{dx}\left[\int_0^{x^2} \cos t \, dt\right]$$

One more challenging example

$$\frac{d}{dx}\left[\int_{5x}^{x^2} \cos t^3 \, dt\right]$$

SF and Calc Lesson 6 WS Name:______________________________

1. What derivative rule is being "undone" by integration by substitution? What two parts are you trying to identify when you rewrite these integrals?

2. Complete the table by identifying the u and du for the integral $\int f(g(x))g'(x)dx$

$\int f(g(x))g'(x)dx$	$u = g(x)$	$du = g'(x)dx$
$\int (5x^2+1)^2(10x)dx$		
$\int x^2\sqrt{x^3+1}dx$		
$\int \frac{x}{\sqrt{x^2+1}}dx$		
$\int \sec 2x \tan 2x dx$		
$\int \tan^2 x \sec^2 x dx$		
$\int \frac{\cos x}{\sin^2 x}dx$		

3. Find the indefinite integral and check the result by mentally taking the derivative.

a. $\int 2(1+2x)^4 dx$

b. $\int -2x\sqrt{9-x^2}dx$

c. $\int x^3(x^4+3)^2 dx$

d. $\int 5x\sqrt[3]{1-x^2}dx$

e. $\int \frac{x}{(1-x^2)^3}dx$

5. Find the indefinite integral:

a. $\int \pi \sin(\pi x)dx$

b. $\int 4x^3 \sin x^4 dx$

c. $\int \cos 6x dx$

d. $\int \frac{1}{\theta^2} \cos \frac{1}{\theta} d\theta$

e. $\int \sec(1-x)\tan(1-x)dx$

f. $\int \tan^4 x \sec^2 x dx$

g. $\int \sqrt{\sec x} \sec x \tan x dx$

h. $\int \csc^2(\frac{\theta}{2})d\theta$

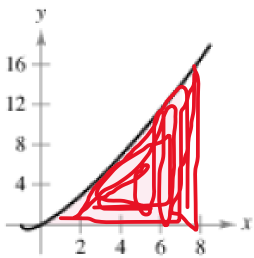
6. Solve the differential equation:

a. $\dfrac{dy}{dt} = 4x + \dfrac{4x}{\sqrt{16 - x^2}}$

b. $\dfrac{dy}{dx} = \dfrac{x^3}{3} + \dfrac{1}{4x^2}$

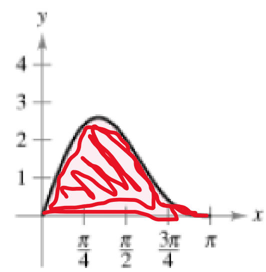
7. Find the area of the region:

$\int_0^7 x\sqrt[3]{x+1}\,dx$



a.

$y = 2\sin x + \sin 2x$



b.

8. The normal monthly rainfall at the Seattle-Tacoma airport can be approximated by the model $R = 3.121 + 2.399\sin(0.524t + 1.377)$ where R is measured in inches and t is the time in months, with t = 1 corresponding to January.

a. Determine the extrema of the function over a one-year period.

b. Use integration to approximate the normal annual rainfall. (Integrate over [0,12])

(Use technology, but show set-up)

9. Evaluate the definite integral:

a. $\int_{-1}^{1} x(x^2+1)^3 dx$

b. $\int_0^4 \frac{1}{\sqrt{2x+1}} dx$

c. $\int_0^{\pi/2} \cos(\frac{2x}{3}) dx$

d. $\int_{\pi/3}^{\pi/2} (x+\cos x) dx$

11. Evaluate the indefinite integral using any rule or method:

a. $\int 2\pi y(8-y^{3/2}) dy$

b. $\int (1+\frac{1}{t})^3 (\frac{1}{t^2}) dt$

c. $\int x\sqrt{1-x} dx$

d. $\int \frac{\csc^2 x}{\cot^3 x} dx$

12. Explain why an odd function h(x) has the following property: $\int_{-a}^{a} h(x)dx = 0$

10. Integrate to find F as a function of x and demonstrate the Second Fundamental Theorem of Calculus by differentiating the result.

a. $F(x) = \int_0^t (t+2)dt$

b. $F(x) = \int_4^x \sqrt{t}dx$

11. Use the Second Fundamental Theorem of Calculus to find F '(x).

a. $F'(x) = \int_{-2}^{x} (t^2 - 2t)dt$

b. $F'(x) = \int_1^x (\frac{t^2}{t^2+1})dt$

c. $F'(x) = \int_x^{x+2} (4t+1)dt$

d. $F'(x) = \int_{-x}^{x} (t^3)dt$

e. $F'(x) = \int_0^{\sin x} \sqrt{t}dt$

f. $F'(x) = \int_0^{x^3} (\sin t^2)dt$

SF and Calc Lesson 6 Mini Quiz   Name: ________________

Evaluate the definite integral using u-substitution.

$\int 6x \sin(x^2)\, dx$

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