

Must show WORK to get FULL CREDIT Answers should be 3 decimal places.

1. Find the indefinite integral.

a.) $\int 2 \cos x + 7 \sin x - x \, dx$

b.) $\int \frac{x^3 + 2x - 4}{\sqrt{x}} \, dx$

2. Evaluate the integrals using the following values: $\int_2^6 f(x) \, dx = 15$, $\int_2^6 g(x) \, dx = 4$, $\int_{-2}^2 f(x) \, dx = -3$,

a.) $\int_6^6 (f(x) - g(x) + 2) \, dx$

b.) $\int_6^2 (g(x) + 3) \, dx$

c.) $\int_2^6 (-2f(x) + 4g(x) + 2) \, dx$

d.) $\int_{-2}^6 (f(x) - 1) \, dx$

3. Evaluate the area of the region defined by the definite integral using geometry formulas.

a.) $\int_0^4 2x + 3 \, dx$

b.) $\int_{-4}^4 \sqrt{16 - x^2} \, dx$

4. Evaluate the following integrals.

a.) $\int_1^3 (2x + 1) \, dx$

b.) $\int \frac{x^2}{\sqrt{x^3 + 3}} \, dx$

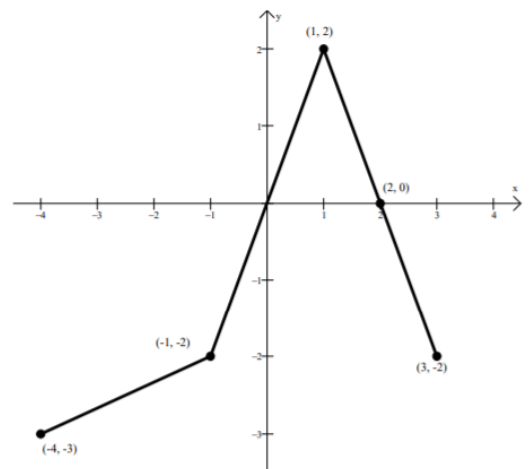
5.) A math student on Mars becomes very angry when he realizes that he will never get to take a math class with Mr. Wenaas. In his anger he throws his calculator with an acceleration of $a(t)=-12t$ given that acceleration is in meters per second squared.

a.) Find an equation for the velocity of the calculator.

b.) Find an equation for the position/height of the calculator given that the calculator was thrown from an initial height of 2 meters, $P(0)=2$, and with an initial velocity 45 meters per second, $V(0)=45$.

6.) Use the second fundamental theorem of calculus to find $F'(x)$ given that $F(x)=\int_{2x}^{x^3} \tan \tan t^2 dt$

7.) Using the given graph find $\int_{-4}^3 f(x)dx$.



AP Calc 4.1-4.5 Test CALC OK

Name: _____

Must show WORK to get FULL CREDIT Answers should be 3 decimal places. Show the work of setting up the integral and taking the antiderivative, but you may use the calculator to evaluate it from there.

1. Estimate the area under the curve using right, left, and either midpoint or trapezoid sums for $f(x)=x^2 - 4$ from $[2,10]$ using 4 equal divisions.

2.) Evaluate the definite integral. $\int_0^2 2x\sqrt{x+2} dx$

3. Find the area between the two curves, $f(x) = 3x^3 + 2x + 2$ and $g(x) = -2x^2 - 4$, on the interval $[1,3]$.

4. Use the mean value theorem for integrals to find a c such that $f(c)$ =average value of $f(x)$ over the interval $[0,3]$ for $f(x)=3x^2 + 2x - 1$

5. What is a slope field?

6. What are some causes of slope fields?

7. Did you enjoy the structure, activities, and connections to real life of this unit more or less than previous units?

8. What would you change about this unit?

9. What was your favorite part of this unit?

Must show WORK to get FULL CREDIT Answers should be 3 decimal places.

1. Find the indefinite integral.

a.) $\int 2 \cos x + 7 \sin x - x \, dx$

$$2 \sin x - 7 \cos x - \frac{1}{2} x^2 + C$$

b.) $\int \frac{x^3 + 2x - 4}{\sqrt{x}} \, dx = \int x^{\frac{5}{2}} + 2x^{\frac{1}{2}} - 4x^{-\frac{1}{2}} \, dx$
$$= \frac{2}{7} x^{\frac{7}{2}} + \frac{4}{3} x^{\frac{3}{2}} + 8x^{\frac{1}{2}} + C$$

2. Evaluate the integrals using the following values: $\int_2^6 f(x) \, dx = 15$, $\int_2^6 g(x) \, dx = 4$, $\int_{-2}^2 f(x) \, dx = -3$,

a.) $\int_6^6 (f(x) - g(x) + 2) \, dx$

$$0$$

b.) $\int_6^6 (g(x) + 3) \, dx = -\int_6^6 (g(x) + 3) \, dx$
$$= -(\underbrace{4}_{\int_2^6 g(x)} + \underbrace{12}_{\int_6^6 3}) = -16$$

c.) $\int_2^6 (-2f(x) + 4g(x) + 2) \, dx$

$$\downarrow$$

$$-30 + 16 + 8 = -6$$

d.) $\int_{-2}^6 (f(x) - 1) \, dx$

$$\downarrow \quad \downarrow$$

$$-3 + 15 - 8 = 4$$

3. Evaluate the area of the region defined by the definite integral using geometry formulas.

a.) $\int_0^4 2x + 3 \, dx$

$$\frac{1}{2} (3 + 11) 4 = 28$$

b.) $\int_{-4}^4 \sqrt{16 - x^2} \, dx$

Semi circle
$$\frac{1}{2} \pi r^2 = \frac{1}{2} \pi 16 = 8\pi$$

4. Evaluate the following integrals.

a.) $\int_1^3 (2x + 1) \, dx$

$$= x^2 + x \Big|_1^3$$

$$= (9 + 3) - (1 + 1)$$

$$= 10$$

or $\frac{1}{2} (3 + 7) 2 = 10$

b.) $\int \frac{x^2}{\sqrt{x^3 + 3}} \, dx$

$$u = x^3 + 3$$

$$du = 3x^2 \, dx$$

$$= \frac{1}{3} \int \frac{1}{\sqrt{u}} \, du = \frac{1}{3} \int u^{-\frac{1}{2}} \, du$$

$$= \frac{1}{3} (2u^{\frac{1}{2}}) + C$$

$$= \frac{2}{3} (x^3 + 3)^{\frac{1}{2}} + C$$

5.) A math student on Mars becomes very angry when he realizes that he will never get to take a math class with Mr. Wenaas. In his anger he throws his calculator with an acceleration of $a(t) = -12t$ given that acceleration is in meters per second squared.

a.) Find an equation for the velocity of the calculator.

$$V(t) = \int a(t) dt = \int -12t dt = -6t^2 + C = V(t)$$

b.) Find an equation for the position/height of the calculator given that the calculator was thrown from an initial height of 2 meters, $P(0) = 2$, and with an initial velocity 45 meters per second, $V(0) = 45$.

$$V(t) = -6t^2 + C$$

$$V(0) = -6(0)^2 + C = 45$$

$$C = 45$$

$$V(t) = -6t^2 + 45$$

$$P(t) = \int V(t) dt$$

$$= -2t^3 + 45t + C$$

$$P(0) = -2(0)^3 + 45(0) + C = 2 \quad C = 2$$

$$P(t) = -2t^3 + 45t + 2$$

6.) Use the second fundamental theorem of calculus to find $F'(x)$ given that $F(x) = \int_{2x}^{x^3} \tan t^2 dt$

$$\frac{d}{dx} \left[\int_{2x}^{x^3} \tan t^2 dt \right] = \tan(x^6) \cdot 3x^2 - \tan(4x^2) \cdot 2$$

$$\text{or } 3x^2 \tan(x^6) - 2 \tan(4x^2)$$

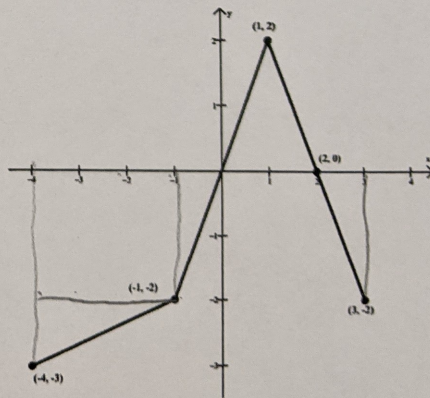
7.) Using the given graph find $\int_{-4}^3 f(x) dx$.

$$-6 - \frac{3}{2} - 1 + 2$$

$$-13\frac{1}{2}$$

or

$$-6.5$$



Graph of f

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1. Estimate the area under the curve using right, left, and either midpoint or trapezoid sums for $f(x) = x^2 - 4$ from $[2, 10]$ using 4 equal divisions.

$$\text{Left} \rightarrow 2(0) + 2(12) + 2(32) + 2(60) = 208$$

$$\text{Right} \rightarrow 2(12) + 2(32) + 2(60) + 2(96) = 400$$

$$\text{Trapezoid} \rightarrow (L+R) \div 2 = 304$$

$$\text{Mid} \rightarrow 2(5) + 2(21) + 2(45) + 2(77) = 296$$

- 2.) Evaluate the definite integral. $\int_0^2 2x\sqrt{x+2} dx$

$$u = x+2 \\ x = u-2$$

$$\int_2^4 2(u-2)\sqrt{u} du = \int_2^4 (2u-4)\sqrt{u} du = \int_2^4 2u^{3/2} - 4u^{1/2} du$$

$$= \left[\frac{4}{5}u^{5/2} - \frac{8}{3}u^{3/2} \right]_2^4 \quad \text{use calc to finish} = \begin{matrix} 7.283 \\ \text{or} \\ 7.284 \end{matrix}$$

3. Find the area between the two curves, $f(x) = 3x^3 + 2x + 2$ and $g(x) = -2x^2 - 4$, on the interval $[1, 3]$.

$$\int_1^3 f(x) - g(x) dx \quad \text{or} \quad \int_1^3 3x^3 + 2x + 2 + 2x^2 + 4 dx \quad \text{or} \quad \int_1^3 3x^3 + 2x^2 + 2x + 6 dx$$

$$= \left[\frac{3}{4}x^4 + \frac{2}{3}x^3 + x^2 + 6x \right]_1^3$$

use calc to finish

$$= 97\frac{1}{3}$$

4. Use the mean value theorem for integrals to find a c such that $f(c) = \text{average value of } f(x) \text{ over the interval } [0, 3]$ for $f(x) = 3x^2 + 2x - 1$

$$\frac{1}{3} \int_0^3 3x^2 + 2x - 1 dx = \frac{1}{3} (x^3 + x^2 - x) \Big|_0^3 = \frac{1}{3} (27 + 9 - 3) = \frac{1}{3} (33) = 11$$

$$11 = 3x^2 + 2x - 1$$

$$0 = 3x^2 + 2x - 12$$

$$\frac{-2 \pm \sqrt{4 + 144}}{6} = x$$

$$x = 1.694$$