Instructions. Answer **any 4 short questions, and any 4 long questions**. Clearly mark which questions you wish to be graded on this sheet, or else 1-4 and 6-9 will be graded. Show all work, and explain your answers clearly. Answers will be graded on correctness and clarity. All solutions should include some explanation.

Shorter questions: (5 points each)

1. Let $\mathbf{n}$ denote an antichain poset with n elements. Write a formula for the number of linear extensions of the ordinal sum poset $\mathbf{n} \oplus \mathbf{m} \oplus \mathbf{p}$.
2. Is the pentagon poset $a \lessdot b \lessdot c \lessdot d, a \lessdot e \lessdot d$ a distributive lattice? Why or why not?
3. Given the following two semistandard Young tableaux, $T = \begin{array}{|c|c|c|} \hline 1 & 3 & 3 \\ \hline 2 & 4 \\ \cline{1-2} \end{array}$ $U = \begin{array}{|c|c|} \hline 1 & 2 \\ \hline 2 \\ \cline{1-1} \end{array}$, find their product, $T * U$, in the plactic monoid.
4. Compute the Schur symmetric function $s_{2,2}$ in three variables x_1, x_2, x_3.
5. Write the basis element of the Specht module corresponding to the standard Young tableau $\begin{array}{|c|c|c|} \hline 1 & 2 & 3 \\ \hline 4 \\ \cline{1-1} 5 \\ \cline{1-1} \end{array}$. You do not need to simplify your answer.

Longer questions: (10 points each)

6. State the fundamental theorem of finite distributive lattices. Outline the proof.
7. For which positive integers is there no connected poset P with exactly n chains? Justify your answer.
8. Let $L(w, k)$ denote the largest sum of the lengths of k disjoint weakly increasing subsequences of w. Prove the following statement: If w and w' are Knuth equivalent words, then $L(w, k) = L(w', k)$.

   Recall the elementary Knuth transformations are:

   (a) $yzx \sim yxz$, $x < y \leq z$
   (b) $xzy \sim zxy$, $x \leq y < z$
9. Let $\Lambda_{\mathbb{Q}}$ denote the ring of homogeneous symmetric functions over $\mathbb{Q}$. Let $\omega : \Lambda_{\mathbb{Q}} \to \Lambda_{\mathbb{Q}}$ be defined by how it acts on the Schur function basis: $\omega(s_\lambda) = s_{\lambda'}$, where λ' is the conjugate of the partition λ.

   (a) What is ω^2? Why?
   (b) Define the hall inner product of symmetric functions by: $\langle s_\lambda, s_\mu \rangle = \delta_{\lambda,\mu}$, where $\delta_{\lambda,\mu} = 1$ if $\lambda = \mu$ and 0 otherwise. Show that ω is an isometry, meaning for symmetric functions $f, g \in \Lambda_{\mathbb{Q}}$, $\langle f, g \rangle = \langle \omega(f), \omega(g) \rangle$.
10. Find the table of characters for the irreducible representations of S_3. Give reasons for each character value; writing the table from memory without justifying your computations will receive no credit.