Department of Mathematics - North Dakota State University
Applied Mathematics Qualifying Exam
May 2008
Subject: MATH 781 Control Theory

1. (25 points) Consider a system with transfer function

$$W(s) = \frac{1}{s^3 + \alpha s^2 + \beta s + 1},$$

where $\alpha > 0$, $\beta > 0$, $\alpha\beta > 1$. Find the maximal number μ such that system is absolutely stable in the class of continuous time-invariant nonlinearities φ satisfying sector condition

$$0 < \frac{\varphi(y)}{y} < \mu \qquad \forall\, y \neq 0.$$

2. (20 points) Given function

$$R(s) = \frac{s+1}{(s-1)(s+2)},$$

find a rational function $X_0 \in H_\infty$ such that $\|R - X_0\|_\infty \leq \|R - X\|_\infty$ for all rational functions $X \in H_\infty$.

3. Assume matrices S and G are Hermitian, $S \geq 0$, $G > 0$, and A is a square matrix. Consider the matrix function $F(P) = G + PA + A^*P - PSP$, where P is Hermitian matrix. Consider a sequence of matrices $\{P_k\}$ satisfying equation

$$G + P_k S P_k + P_{k+1}(A - SP_k) + (A - SP_k)^* P_{k+1} = 0.$$

(i) (20 points) Assume $P_0 \geq 0$, and $A - SP_0$ is Hurwitz. Show that P_1 is well defined, $P_1 \geq 0$, and $A - SP_1$ is Hurwitz.

(ii) (5 points) Assume $P_0 \geq 0$, and $A - SP_0$ is Hurwitz. Show that P_k is well defined, $P_k \geq 0$, and $A - SP_k$ is Hurwitz for all $k > 0$.

(iii) (20 points) Assume $F(P_0) \leq 0$. Show that $F(P_1) \leq 0$. Assume additionally that matrix $A - SP_0$ is Hurwitz. Prove that $P_1 - P_0 \leq 0$.

(iv) (10 points) Assume $P_0 \geq 0$, $F(P_0) \leq 0$, and matrix $A - SP_0$ is Hurwitz. Show that the sequence $\{P_k\}$ is convergent: $P_k \to \bar{P}$, matrix $\bar{P}$ satisfies the Riccati equation $F(\bar{P}) = 0$, and matrix $A - S\bar{P}$ is Hurwitz.