Department of Mathematics - North Dakota State University
Applied Mathematics Qualifying Exam
August 2008
Subject: MATH 760 Ordinary Differential Equations

1. (15 points) Prove that for any pair of numbers (t_0, y_0) a solution to the IVP $y' = t^3 - y^3$, $y(t_0) = y_0$ may be extended to the interval $[t_0, \infty)$.

2. (15 points) What is the smallest natural number n such that there exists a smooth function f with the following property: ODE $y^{(n)} = f(t, y, \dots, y^{(n-1)})$ has solutions $y_1 = t - t^3/6$, and $y_2 = \sin t$?

3. (15 points) Find all values of parameters a, b such that the equation $y^{IV} + ay''' + y'' + by' + y = 0$ is asymptotically stable.

4. (20 points) Find all values of a parameter α such that system $y' = A(t)y$ is Lyapunov stable, where

$$A(t) = \begin{pmatrix} 0 & \alpha \\ 0 & 0 \end{pmatrix} \; if \;\; 0 \leq t < 2, \qquad A(t) = \begin{pmatrix} 0 & 0 \\ -\alpha & 0 \end{pmatrix} \; if \;\; 2 \leq t < 4, \qquad A(t+4) \equiv A(t).$$

5. (15 points) Assume functions p, q are continuous. Let y_1, y_2 be linear independent solutions of equation

$$y'' + p(t)y' + q(t)y = 0.$$

Show that function $\frac{y_1}{y_2}$ has no points of local maximum.

6. (20 points) Find all the parameters $a \in R$ for which the equation

$$y' = (a + \cos^2 t)y + 1$$

has exactly one periodic solution.