Department of Mathematics - North Dakota State University
Applied Mathematics Qualifying Exam
August 2008
Subject: MATH 781 Control Theory

1. (20 points) The transfer function of system is equal to

$$W(s) = \frac{1}{s^3 + s^2 + s + \gamma},$$

where $\gamma \in (0,1)$. Find the maximal number μ such that system is absolutely stable in the class of differentiable nonlinearities φ satisfying inequalities $0 < \frac{\varphi(s)}{s} < \mu$ for all s.

2. (15 points) Consider a system with the transfer function

$$W(s) = \frac{a}{s^2 + bs + 1},$$

where $a > 0$, $b > 0$. Using the circle criterion, find a positive numbers μ such that system is absolutely stable in the class of time-varying nonlinearities φ satisfying the sector condition

$$0 \leq \frac{\varphi(y,t)}{y} \leq \mu$$

for all $y \neq 0$, t.

3. (10 points) Find all parameters a, b, c such that system

$$\dot{x} = \begin{pmatrix} 0 & b \\ c & 1 \end{pmatrix} x + \begin{pmatrix} 1 \\ 0 \end{pmatrix} u,$$

$$y = (a,\ 0)\, x$$

(i) is controllable,
(ii) is stabilizable,
(iii) is observable.

4. (25 points) Assume A, S, G are $n \times n-$matrices, S and G are Hermitian, $S \leq 0$. Assume H_0 is the anti-Hurwitz solution of the algebraic Riccati equation $H_0 A + A^* H_0 + G - H_0 S H_0 = 0$. Assume a Hermitian matrix function $H(\cdot)$ satisfies the differential Riccati equation

$$\dot{H} + HA + A^*H + G - HSH = 0.$$

Denote $A_{H_0} = A - SH_0$.
(i) Prove the following equality:

$$\dot{H} + (H - H_0)A_{H_0} + A^*_{H_0}(H - H_0) - (H - H_0)S(H - H_0) = 0.$$

(ii) Assume $H(0) > H_0$. Show that $H(t) > H_0$ for all $t > 0$, and $H(t) \to H_0$ as $t \to \infty$.

5. (15 points) Given function $R(s) = \frac{s+2}{s^2-1}$, compute the minimum of $\|R - X\|_\infty$ over all rational functions $X \in H_\infty$.

6. (15 points) For functions $T_1 = \frac{s-1}{s+1}$, $T_2 = \frac{1}{s+3}$ find $\min \|T_1 - T_2 Q\|$ over the set of rational functions $Q \in H_\infty$.