Problems for Preliminary Exam
Applied Mathematics, ODE
January 2023

1. For which pairs of positive numbers k, ω equation

$$y'' + k^2 y = \sin \omega t$$

has at least one periodic solution?

2. Find all numbers a for which the Boundary Value Problem

$$y'' + ay = 1, \qquad y(0) = 0, \;\; y(1) = 0$$

has no solutions.

3. Assume y is a solution on $[a, b]$ of equation

$$y'' + q(t)y = 0$$

with $q(t) \leq 0$ for all t. Assume $y(a) = 0$. Prove that function y' does not change sign on $[a, b]$.

4. Consider a linear system

$$\begin{aligned} \dot{x}_1 &= a_{11}(t)x_1 + a_{12}(t)x_2 \\ \dot{x}_2 &= a_{21}(t)x_1 + a_{22}(t)x_2, \end{aligned}$$

where functions $a_1, a_{12}, a_{21}, a_{22}$ are continuous. Assume $a_{11}(t) + a_{22}(t) \to b > 0$ as $t \to \infty$. Prove that system is unstable.

5. Find all values of numbers a, b for which equation

$$y'''' + 2y''' + 4y'' + ay' + b = 0$$

is asymptotically stable.

6. Find the smallest positive number T such that equation

$$\ddot{y} - 2\dot{y} = 8 \sin^2 t$$

has a solution satisfying boundary conditions $\dot{y}(0) = -1$, $\dot{y}(T) = -1$.

7. Does there exist an unbounded on $[0, \infty)$ solution of equation

$$\ddot{y} = 4y - 4y^3?$$