Topology/Geometry Preliminary Exam.
Math 746

August 2021

Problem 1. Let X be a set of cardinality three.
a) How many topologies are there on X?
b) Up to a homeomorphism, how many topological spaces are there on X?

Problem 2. Find an explicit embedding of Klein Bottle into $\mathbb{R}^4$.

Problem 3. Let X, Y be topological spaces and $p : X \to Y$ be a finite-sheeted covering map.
a) If X is compact, then is it true that Y is compact?
b) If Y is compact, then is it true that X is compact?

Problem 4. Prove that $\mathrm{SO}(3, \mathbb{R})$ is homeomorphic to $\mathbb{R}\mathrm{P}^3$.

Problem 5. Let $n \geq 1$.
a) Find finite cell divisions of $\mathbb{R}\mathrm{P}^n$ and $\mathbb{C}\mathrm{P}^n$.
b) Compute homology groups of $\mathbb{R}\mathrm{P}^n$ and $\mathbb{C}\mathrm{P}^n$.
c) Find $\pi_1(\mathbb{R}\mathrm{P}^n)$ and $\pi_1(\mathbb{C}\mathrm{P}^n)$.