

Preliminary Exam, Math 746

January 2022

Problem 1. Give an example of a topological space which is

- a) connected but not path-connected
- b) path connected but not locally path connected.

Problem 2. What compact 2-manifolds does one obtain by pairwise gluing the sides of a hexagon?

Problem 3. a) Find a topological space X such that $H_1(X) \cong \mathbb{Z}^{2022}$, $H_2(X) \cong \mathbb{Z}$ and $H_n(X) \cong 0$ for all $n \geq 3$.

b) Is there a manifold satisfying conditions of part a)?

Problem 4. a) Let $n \geq 2$ be a positive integer. Is it true that $\mathbb{S}^1 \vee \mathbb{S}^1$ admits a connected n -sheeted covering space?

b) Describe the universal covering space of $\mathbb{S}^1 \vee \mathbb{S}^1$.

Problem 5. Let X be the sphere of radius 2 centered at $(0, 0, 0)$ and Y be the sphere of radius 2 centered at $(1, 0, 0)$ in \mathbb{R}^3 . Compute all homology groups and the fundamental group of $X \cup Y$ (viewing $X \cup Y$ as a subspace of \mathbb{R}^3).